GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

CALCULUS 11

CMS 1201

Semester Two 2025/2026 : Main Examination

This examination paper consists of 3 printed pages, exclusive of this cover page.

Date:	**JUNE 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Ms B. Kwirira**

INSTRUCTIONS

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY TWO** questions from **Section B** each question carries **30 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

ADDITIONAL MATERIALS

- Calculator (non-programmable)
- Pens and Pencils

SECTION A: Answer ALL questions [40].

A1. (a) Define what is **meant** by saying two vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular. [2]

(b) Let $\mathbf{A} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j}$. Find a unit vector orthogonal to both $\mathbf{A}$ and $\mathbf{B}$. [6]

A2. The epicycloid C is given by $\mathbf{r}(x) = (5\cos x - \cos 5x)\mathbf{i} + (5\sin x - \sin 5x)\mathbf{j}$, $0 \leq x \leq 2\pi$. Find the intervals in which C is smooth. [6]

A3. (a) Define a smooth curve. [2]

(b.) Determine the nature of stationery points for the function

$$f(x, y) = x^2 - 2xy + y^2 - 2x + 2y + 4.$$

[4]

A4. Let $f(x, y, z) = x^2 \ln(xyz) \sin(yz)$. Find the gradient of the function $f(x, y, z)$. [8]

A5. Evaluate $\int_1^4 \int_0^1 \int_0^x 2ze^{-x^2} dydxdz$. [6]

A6. Determine whether the series $\sum_{m=0}^{\infty} \frac{m^2 2^{m+1}}{3^m}$ converges or diverges. [6]

Section B: Answer any TWO questions (60 marks)

A7. (a) Find a set of parametric equations of the line that passes through the points $P(-2,1,0)$ and $Q(1,3,5)$. [6]

(b) Let $A=(1,1,1)$, $B=(0,3,1)$ and $C=(-1,2,4)$ be points on the plane P_1. Find vectors **AB** and **AC**. Hence or otherwise, find the equation of the plane P_1. [10]

(c) Let P_2 be a plane with equation $2x+3y+4z=5$. Find the line of intersection between P_1 in question $B7(b)$ and the plane P_2. [6]

(d) A particle moves along the curve C given by $\mathbf{r}(t)=2\sin\frac{t}{2}\mathbf{i}+2\cos\frac{t}{2}\mathbf{j}$. Find in terms of t the velocity vector, acceleration vector and the speed of the particle. [3,3,2]

A8. (a) (i) Define what is **meant** by saying the function $f(x,y)$ converges to l as (x,y) tends to a point (x_0,y_0). [2]

(ii) Show that $\lim\limits_{(x,y)\to(2,3)} x^2+2y=10$. [8]

(b) Show that the function

$$f(x,y) \;=\; \begin{cases} \left(\dfrac{x^2-y^2}{x^2+y^2}\right)^2, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

is not continuous at the point $(0,0)$. [4]

(c) Let $u=x+y$ and $v=xy$. Find $\dfrac{\partial x}{\partial u}$, $\dfrac{\partial x}{\partial v}$, $\dfrac{\partial y}{\partial u}$ and $\dfrac{\partial y}{\partial v}$. [8]

(d) Find the equation of the tangent plane to the hyperboloid given by

$$z^2=2x^2+2y^2+12$$

at the point $(1,-1,4)$. [8]

A9. (a) The spherical coordinates are given by $x=r\cos\theta\sin\phi$, $y=r\sin\theta\sin\phi$ and $z=r\cos\phi$. Show that the Jacobian from (x,y,z) to (r,θ,ϕ) is given by $-r^2\sin\phi$. [10]

(b) Sketch the region of integration R of the double integral $\int\int_R (2x^2y^{-2}+2y)dydx$, where R is bounded by $1\leq x\leq 2$ and $1\leq y\leq x$.
Hence or otherwise evaluate $\int\int_R (2x^2y^{-2}+2y)dydx$. [10]

(c) Find $\int\int_R (x^2+y^2)dxdy$, where R is the region bounded by $xy=1$, $xy=3$, $x^2-y^2=1$ and $x^2-y^2=4$. [10]