GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

NUMERICAL METHODS AND ORDINARY DIFFERENTIAL EQUATIONS

ESG 2212

Examination Paper

APRIL 2025

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. R. G. Moyo**

INSTRUCTIONS

Candidates should answer **ALL** questions in section A and **ANY THREE** questions in section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Define the following terms

(i) Truncation error [2]

(ii) Numerical Ill-conditioning [2]

(iii) Runge-Kutta Method of Order Four (RK4s) [2]

A2. (i) Outline the steps followed when solving a non-linear equation using the bisection method [4]

(ii) Use the Bisection Method to find the root of $f(x) = \cos x - \sqrt{x}$ lying between [0,1]. Use a tolerance of $\epsilon = 0.01$ and give your answer correct to 5 decimal places [7]

A3. (i) Use the Simpson's rule with $n = 10$ to obtain an approximation to $\int_0^2 xe^{3x}dx$ correct to 5 decimal places. [6]

(ii) Consider the following table

x	1.3	1.4	1.45	1.5	1.55	1.6
$f(x)$	4.77	5.677	6.182	6.732	7.3028	7.9248

Evaluate

(a) $f'(1.4)$ using the forward difference method [3]

(b) $f'(1.5)$ using the central difference method for $h = 0.1$ [3]

(c) $f''(1.6)$ [4]

A4. Use the following values to construct a third degree Lagrange approximation to $f(1.25)$ correct to 5 decimal places : [7]

$f(1.0) = 1.00000$
$f(1.1) = 1.23368$
$f(1.2) = 1.55271$
$f(1.3) = 1.99372$

SECTION B (60 marks)
Answer ANY THREE questions from this section.

B5. (i) State the three elementary row equations used when solving a system of linear equations. [3]

(ii) Given the following system of equations

$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$
$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$
$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$
$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$

(a) Write down the system in the form $\mathbf{A}\overline{x} = b$ [3]

(b) Find the inverse of the matrix $\mathbf{A}$ using elementary row operations [10]

(c) Hence use the inverse of matrix $\mathbf{A}$ to solve the system [4]

B6. Given an initial value problem (IVP) defined by $y' = 2xy + y^2$, $y(1) = 1$

(i) Evaluate $y(1.3)$ using the Taylor's third order method. Use $h = 0.1$ [12]

(ii) Calculate $y(1.2)$ using Runge-Kutta Method of order 4 (RK4) using $h = 0.1$ [8]

B7. (i) Construct a linear interpolation function for the ponts (3, 2) and (5, 8) [2]

(ii) Differentiate between Interpolation and Extrapolation [2]

(iii) (a) Construct the Newton polynomial function $P_3(x)$ for the points $(x, \cos x)$ correct to 6 decimal places using $x = 0, 1, 2, 3$. [5]

(b) Evaluate $P_3(2.785)$ [3]

(c) Hence calculate the error in estimating the value of $\cos 2.785$ [2]

(iv) Write a Matlab code to solve an initial value problem (IVP), $y' = \frac{x-y}{2}$, $y(0) = 1$ on $[0, 100]$ [6]

B8. (i) Discuss the application of error analysis in Geomatics and Surveying [5]

(ii) Solve the following system of equations using the Naive-Gaussian Elimination. [15]

$x_1 - 4x_2 + 4x_3 + 7x_4 = 4$
$2x_2 - x_3 = 5$
$2x_1 + x_2 + 2x_3 + 4x_4 = 2$
$2x_1 - 3x_2 + 2x_3 - 5x_4 = 9$

END OF QUESTION PAPER