GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

General Topology

CMS 2204

Semester Two 2025/2026 : Examination

This examination paper consists of 3 printed pages, exclusive of this cover page.

Date:	**JUNE 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Mr. M. Ndlovu**

INSTRUCTIONS

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY THREE** questions from **Section B** each question carries **20 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

ADDITIONAL MATERIALS

- Calculator (non-programmable)
- Pens and Pencils

SECTION A: Answer ALL questions [40].

A1. (a) Compare and contrast Geometry and Topology. **(4 Marks)**

(b) Check continuity of

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases}$$

at $x = 0$ using the Squeeze Theorem. **(6 Marks)**

A2. (a) Complete this statement:
A subset of $\mathbb{R}$ *is closed if and ..... if its complement is .....* **(2 Marks)**

(b) Show that the interval $[1, 3] \subset \mathbb{R}$ is a closed set in the standard topology on $\mathbb{R}$. **(5 Marks)**

(c) Define a topology on $X = \{0, 1, 2\}$. **(6 Marks)**

A3. (a) Show that the projection map

$$\pi : \mathbb{R}^2 \to \mathbb{R}, \qquad \pi(x, y) = x,$$

is continuous when $\mathbb{R}^2$ has the standard topology from open balls. **(6 Marks)**

(b) Does a cylinder

$$C = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1,\ 0 \leq z \leq 1\},$$

form a topological space. **(6 Marks)**

A4. Define a digital topological space. **(5 Marks)**

SECTION B: Answer ANY three questions [60].

B5. (a) Prove that the sphere $S^2 \subset \mathbb{R}^3$ is compact. **(6 Marks)**

(b) (i) State the Hausdorff Property. **(4 Marks)**

(ii) Prove that $\mathbb{R}^2$ is Hausdorff. **(5 Marks)**

(iii) If $f : S \to T$ is continuous and injective, and T is Hausdorff, prove that S is Hausdorff. **(5 Marks)**

B6. (a) Define what it means for two topological spaces to be homeomorphic. **(4 Marks)**

(b) Prove that any open interval

$$(a, b) \subset \mathbb{R}$$

is homeomorphic to $\mathbb{R}$. **(10 Marks)**

(c) Prove that the real projective line $\mathbb{R}P^1 \cong S^1$. **(6 Marks)**

B7. (a) Define the quotient topology on $X/\sim$. **(4 Marks)**

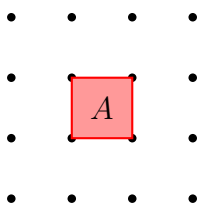
(b) Construct the sphere S^2 step by step from the disk D^2 by collapsing the boundary:

$$S^2 \cong D^2/\partial D^2.$$

(12 Marks)

(c) Explain geometrically why this quotient identification yields a sphere. **(4 Marks)**

B8. The diagram below represents a digital object in $\mathbb{Z}^2$ using 4-adjacency:



Answer all parts below.

(a) Define the following concepts clearly:

(i) A digital adjacency relation on $\mathbb{Z}^n$. **(2 Marks)**

(ii) A digitally continuous function. **(2 Marks)**

(iii) A digital path in (X, κ). **(2 Marks)**

(b) Let $X = \mathbb{Z}^2$ with 4-adjacency, and consider the subset

$$A = \{(1,1), (2,1), (2,2), (1,2)\},$$

illustrated in the diagram.

(i) Determine whether A is digitally connected under 4-adjacency. **(5 Marks)**

(ii) Justify your answer using adjacency relations. **(1 Marks)**

(c) Let $f : (\mathbb{Z}^2, 4) \to (\mathbb{Z}^2, 8)$ be defined by

$$f(x, y) = (x, x + y).$$

Determine whether f is digitally continuous.
(Hint: Check whether 4-adjacent points map to 8-adjacent points.) **(10 Marks)**