GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS III

EMN/EMG 2101

Examination Paper

SEPTEMBER 2024

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. R. G. Moyo**

INSTRUCTIONS

Candidates should answer **ALL** questions in section A and **THREE** questions in section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Define the following terms as they are used in Numerical methods

(i) Numerical ill-conditioning [2]

(ii) Truncation error [2]

(iii) Ordinary differential equation [2]

A2. (i) What is the difference between precision and accuracy. [4]

(ii) Write down the convergence criterion for the Newton-Raphson formula. [2]

(iii) Verify that when the Newton-Raphson method is used to compute $\sqrt{D}$, then the sequence of iterates is defined by
$x_{n+1} = \frac{1}{2}(x_n + \frac{D}{x_n})$
Hence use an iterative process to evaluate $(70)^{\frac{1}{2}}$ correct to 6 decimal places starting with $x_0 = 5.575$ [3,4]

A3. Use Simpson's Rule to integrate the points represented by the table below

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.669	1.811	1.971	2.151	2.325	2.577	2.828	3.107

(i) using $h = 0.1$ [4]

(ii) using $h = 0.2$ [4]

A4. Given that $f(x) = 2xe^{x^2}$, find approximations to $f''(0.4)$ using $h = 0.1$ and $h = 0.001$. Compare the calculated values with the true value of $f''(0.4)$ and hence comment on the effect of changing step size. [13]

SECTION B (60 marks)
Answer ANY THREE questions from this section.

B5. (i) State the three elementary row equations used when solving a system of linear equations. [3]

(ii) Given the following system of equations

$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$
$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$
$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$
$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$

(a) Write down the system in the form $\mathbf{A}\overline{x} = b$ [3]
(b) Find the inverse of the matrix $\mathbf{A}$ using elementary row operations [10]
(c) Hence use the inverse of matrix $\mathbf{A}$ to solve the system [4]

B6. Given an initial value problem (IVP) defined by $y' = 2xy + y^2$, $y(1) = 1$

(i) Evaluate $y(1.3)$ using the Taylor's method. Use $h = 0.1$ [12]
(ii) Calculate $y(1.2)$ using Runge-Kutta Method of order 4 (RK4) using $h = 0.1$ [8]

B7. (i) Construct a linear interpolation function for the ponts (3, 2) and (5, 8) [2]
(ii) Differentiate between Interpolation and Extrapolation [2]
(iii) (a) Construct the Newton polynomial function $P_3(x)$ for the points $(x, \cos x)$ correct to 6 decimal places using $x = 0, 1, 2, 3$. [5]
(b) Evaluate $P_3(2.785)$ [3]
(c) Hence calculate the error in estimating the value of $\cos 2.785$ [2]
(iv) Write a Matlab code to solve an initial value problem (IVP), $y' = \frac{x-y}{2}$, $y(0) = 1$ on $[0, 100]$ [6]

B8. (i) Discuss the application of numerical analysis in mining engineering [5]
(ii) Solve the following system of equations using the Gauss Jordan Elimination. [15]

$x_1 - 4x_2 + 4x_3 + 7x_4 = 4$
$2x_2 - x_3 = 5$
$2x_1 + x_2 + 2x_3 + 4x_4 = 2$
$2x_1 - 3x_2 + 2x_3 - 5x_4 = 9$

END OF QUESTION PAPER