GWANDA STATE UNIVERSITY

CMS 1102

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

LINEAR MATHEMATICS I

EPOCH MINE CAMPUS: FILABUSI

Mr M NDLOVU

SEPTEMBER 2024: SPECIAL EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

Instruments and Materials

- Non-Programmable Calculator.

SECTION A: Answer ALL questions [40].

A1. Define the following terms

(a) Vector [2]

(b) Kernel [2]

(c) Triangular matrix [2]

(d) An algebra [2]

(e) Free variable [2]

A2. Let $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find

(a) $\cos\theta$, where θ is the angle between u and v [3]

(b) $\mathrm{proj}(u, v)$, the projection of u onto v [3]

(c) $d(u, v)$, the distance between u and v. [2]

A3. (a) Prove the *Minkowski* inequality $\|u + v\| \leq \|u\| + \|v\|$ [6]

(b) Find a parametric representation of the line L in $\mathbb{R}^4$ passing through $P(4, 2, 3, 1)$ in the direction of $u = [2, 5, -7, 8]$. [4]

(c) Find the transpose of $B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and comment on your answer. [4]

A4. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $g(x) = x^2 + 2x + 11$.

Compute

(a) $\mathrm{daig}(A)$ [1]

(b) $\mathrm{tr}(A)$ [1]

(c) A^2 [2]

(d) $g(A)$ [4]

SECTION B: Answer ANY three questions [60].

B5. (a) Write down the conjugate of $A = \frac{1}{3}\begin{bmatrix} 1-2i & 2i \\ -2i & -1-2i \end{bmatrix}$. [2]

(b) Find the LU factorization of $\begin{bmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}$. [8]

(c) Solve the system of equations

$$\begin{aligned} 3y + 2x &= z + 1 \\ 3x + 2z &= 8 - 5y \\ 3z - 1 &= x - 2y \end{aligned}$$

[10]

B6. (a) Let $V = \mathbb{R}^3$. Show that W is not a subspace of V, where

$$W = \{(a, b, c) : a \geq 0\}.$$

[3]

(b) Express the polynomial $v = t^2 + 4t - 3$ in $P(t)$ as a linear combination of the polynomials

$$p_1 = t^2 - 2t + 5, \quad p_2 = 2t^2 - 3t, \quad p_3 = t + 1.$$

[9]

(c) Consider the vector space $P(t)$ with inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$.

Apply the Gram– Schmidt algorithm to the set $\{1,\ t,\ t^2\}$ to obtain an orthogonal set $\{f_0,\ f_1,\ f_2\}$ with integer coefficients. [8]

B7. (a) Does $(1,1,1)$ and $(1,0,1)$ form basis of $\mathbb{R}^3$. [2]

(b) Suppose $F:\mathbb{R}^3 \to \mathbb{R}^2$ is defined $F(x,y,z) = (|x|,\ y+z)$.
Show that F is not linear. [2]

(c) Let $F:\mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by

$$F(x,y,z,t) = (x-y+z+t,\ x+2z-t,\ x+y+3z-3t)$$

Find a basis and the dimension of

(i) the image of F [8]

(ii) the kernel of F [8]

B8. (a) Define a transition matrix P (or change of basis matrix). [2]

(b) Consider the following two bases of $\mathbb{R}^2$:

$$S = \{u_1, u_2\} = \{(1,2),\ (3,5)\} \text{ and } S' = \{v_1, v_2\} = \{(1,-1),\ (1,-2)\}$$

(i) Find the change-of-basis matrix P from S to the "new" basis S'. [7]

(ii) Find the change-of-basis matrix Q from the "new" basis S' back to the "old" basis S. [7]

(c) Find $\det(\mathcal{M})$, where $\mathcal{M} = \begin{bmatrix} 3 & 4 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 & 0 \\ 0 & 9 & 2 & 0 & 0 \\ 0 & 5 & 0 & 6 & 7 \\ 0 & 0 & 4 & 3 & 4 \end{bmatrix}$ [4]

END OF QUESTION PAPER