GWANDA STATE UNIVERSITY

Faculty of Computational Sciences

DEPARTMENT OF MATHEMATICS AND STATISTICS

Statistical Inference 1

CMS 1104

Examination Paper

NOVEMBER 2024

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: 100

Examiner's Name: Mr. E. Utete

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

Graph papers

Statistical Tables

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SECTION A : Answer ALL Questions 40 marks

A1 (a) State three properties of the Maximum Likelihood Estimator. [3]

(b) State the Central Limit Theorem. [3]

(c) Give two reasons for using R-programming over Python for data analysis. [2]

(d) Differentiate

i. unbiased estimator and minimum variance unbiased estimator. [3]

ii. power of a statistical test and size of a statistical test. [3]

A2 Eight measurements were made on the inside diameter of forged piston rings used in an automobile engine. The data (in millimeters) are
74.001, 74.003, 74.015, 74.000, 74.005, 74.002, 74.005, and 74.004.

(a) Calculate the sample mean. [3]

(b) Calculate the sample variance and sample standard deviation. [6]

A3 A group of wine enthusiasts taste-tested a pinot noir wine from Oregon. The evaluation was to grade the wine on a 0 to 100 point scale. The results follow:

94 90 92 91 91 86 89 91 91 90 90 93 87 90 91 92 89 86 89 90 88 95 91 88 89 92 87 89
95 92 85 91 85 89 88 84 85 90 90 83

(a) Construct a stem-and-leaf diagram for this data and comment on any important features that you notice. [6]

(b) Use the stem-and-leaf diagram to find median, upper quartile and lower quartile. [6]

(c) A wine rated above 90 is considered truly exceptional. What proportion of the taste-tasters considered this particular pinot noir truly exceptional? [5]

SECTION B : Answer THREE QUESTIONS only : 60 marks

B4 (a) Suppose that X has a discrete uniform distribution

$$f(x) = \begin{cases} \frac{1}{3}; & x = 1; 2; 3 \\ 0, & otherwise \end{cases}$$

A random sample of $n = 36$ is selected from this population. Find the probability that the sample mean is greater than 2.1 but less than 2.5, assuming that the sample mean would be measured to the nearest tenth. [7]

(b) A random sample of size $n_1 = 16$ is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean 70 and standard deviation 12. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means.

Find

i. The probability that $\bar{X}_1 - \bar{X}_2$ exceeds 4. [7]

ii. The probability that $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$. [6]

B5 Let X be a random variable with the following probability distribution:

$$f(x) = \begin{cases} \theta(1-\theta)^{x-1}, & 0 < \theta < 1, \quad x = 1; 2; 3... \\ \\ 0, & otherwise \end{cases}$$

(a) Write down the likelihood function of θ [3]

(b) Find the maximum likelihood estimator of θ, based on a random sample of size n. [5]

(c) Find the maximum likelihood estimator of $\psi = \frac{1}{\theta}$. [5]

(d) Determine the bias, variance and the mean squared error of the maximum likelihood estimator of ψ . [4]

(e) Is the maximum likelihood estimator of ψ unbiased? Is it consistent? [3]

B6 Suppose $X_1; X_2; ...; X_n$ is a random sample from the $Exp(\lambda)$ distribution. Consider the following estimators for $\theta = \frac{1}{\lambda}$

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \quad and \quad \hat{\theta}_2 = \frac{1}{n+1}\sum_{i=1}^{n} X_i$$

(a) Find the biases of $\hat{\theta}_1$ and $\hat{\theta}_2$. [6]

(b) Find the variances of $\hat{\theta}_1$ and $\hat{\theta}_2$. [6]

(c) Find the mean squared errors of $\hat{\theta}_1$ and $\hat{\theta}_2$. [5]

(d) Which of the two estimators ($\hat{\theta}_1$ or $\hat{\theta}_2$) is better and why? [3]

B7 An engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is approximately normally distributed with $\sigma = 60$ *psi*. A random sample of 12 specimens has a mean tensile strength of $\bar{x} = 3250$ *psi*.

(a) Test the hypothesis that mean strength is 3500 *psi*. Use $\alpha = 0.01$. [10]

(b) What is the smallest level of significance at which you would be willing to reject the null hypothesis? [5]

(c) Explain how you could answer the question in part B7(a) with a two-sided confidence interval on mean tensile strength.

[5]