



# **GWANDA STATE UNIVERSITY**

**Faculty of Computational Sciences**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

## **Probability Theory II**

**CMS 2101**

**Examination Paper**

**NOVEMBER 2024**

This examination paper consists of 3 printed pages

**Time Allowed: 3 hours**

**Total Marks: 100**

**Examiner's Name: Mr. E. Utete**

### **INSTRUCTIONS**

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

### **ADDITIONAL REQUIREMENTS**

Scientific calculator

Graph papers

Statistical Tables

**SECTION A : Answer ALL Questions 40 marks**

**A1** What is the difference between covariance and correlation. [4]

**A2** Given that

$$f(x, y) = ce^{-2x-3y} \quad \text{for } 0 < x \text{ and } 0 < y < x$$

(a) Determine the value of  $c$  that makes the function,  $f(x, y)$  a joint probability density function. [6]

(b) Determine

i.  $E(X)$  [5]

ii.  $E(Y)$  [5]

iii.  $P(X < 1, Y < 2)$  [6]

**A3** If  $X$  and  $Y$  are independent, normal random variables with  $E(X) = 0$ ,  $V(X) = 4$ ,  $E(Y) = 10$ , and  $V(Y) = 9$ . Determine the following:

(a)  $E(2X + 3Y)$  [4]

(b)  $V(2X + 3Y)$  [5]

(c)  $P(2X + 3Y < 40)$  [5]

**SECTION B : Answer THREE QUESTIONS only : 60 marks**

**B4** A continuous random variable  $X$  has the following probability distribution:

$$f(x) = 4xe^{-2x} \quad x > 0$$

(a) Find the moment generating function for  $X$ . [10]

(b) Find the mean of  $X$ . [4]

(c) Find the variance of  $X$ . [6]

- B5** A manufacturer of electroluminescent lamps knows that the amount of luminescent ink deposited on one of its products is normally distributed with a mean of 1.2 grams and a standard deviation of 0.03 grams. Any lamp with less than 1.14 grams of luminescent ink will fail to meet customer's specifications. A random sample of 25 lamps is collected and the mass of luminescent ink on each is measured.
- (a) What is the probability that at least 1 lamp fails to meet specifications? [5]
  - (b) What is the probability that 5 lamps or fewer fail to meet specifications? [8]
  - (c) What is the probability that all lamps conform to specifications? [7]
- B6** (a) Determine the value for  $c$  and the covariance and correlation for the joint probability density function  $f_{XY}(x; y) = cxy$  over the range  $0 < x < 3$  and  $0 < y < x$ . [10]
- (b) Let  $X$  and  $Y$  represent two dimensions of an injection molded part. Suppose  $X$  and  $Y$  have a bivariate normal distribution with  $\sigma_X = 0.04$ ,  $\sigma_Y = 0.08$ ,  $\mu_X = 3.00$ ,  $\mu_Y = 7.70$ , and  $\rho_{XY} = 0$ . Determine  $P(2.95 < X < 3.05; 7.60 < Y < 7.80)$ . [10]
- B7** The yield in pounds from a day's production is normally distributed with a mean of 1500 pounds and standard deviation of 100 pounds. Assume that the yields on different days are independent random variables.
- (a) What is the probability that the production yield exceeds 1400 pounds on each of five days next week? [10]
  - (b) What is the probability that the production yield exceeds 1400 pounds on at least four of the five days next week? [10]