# GWANDA STATE UNIVERSITY

## FACULTY OF COMPUTATIONAL SCIENCES

### DEPARTMENT OF MATHEMATICS AND STATISTICS

### COMPLEX ANALYSIS

## CMS 2206

**Semester Two 2025/2026 : Examination**

This examination paper consists of 4 printed pages, exclusive of this cover page.

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| **Date:** | **JUNE 2026** |
| **Total:** | **100 Marks** |
| **Time:** | **3 hours** |
| **Examiner's Name:** | **Mr. M. Mpofu** |

**INSTRUCTIONS**

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY THREE** questions from **Section B** each question carries **20 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

**ADDITIONAL MATERIALS**

- Calculator (non-programmable)
- Pens and Pencils

**SECTION A: Answer ALL questions [40].**

**A1.** (a) Let $z = 1 + 2i$ and $w = 2 - i$. Compute

(i) $Re(w^2 + w)$ **(2 Marks)**

(ii) $z^2 + \bar{z} + iw$ **(2 Marks)**

(b) Prove the triangle inequality for sums:

$$\left| \sum_{k=1}^{n} z_k \right| \leq \sum_{k=1}^{n} |z_k|$$

**(5 Marks)**

**A2.** Suppose the complex function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain $D$. Prove the functions $u(x, y)$ and $v(x, y)$ are harmonic in $D$. **(6 Marks)**

**A3.** By applying the Jordan's Lemma evaluate

$$\int_{-\infty}^{\infty} \frac{(x + b) \sin x}{x^2 + a^2} dx$$

**(5 Marks)**

**A4.** (a) Expand $f(z) = \dfrac{1}{z(z - 1)}$ in a Laurent series that satisfy

(i) $0 < |z| < 1$ **(3 Marks)**

(ii) $1 < |z|$ **(3 Marks)**

(b) Find the circle and radius of convergence for the series

$$\sum_{n=0}^{\infty} \frac{1}{(1 - 2i)^{k+1}} (z - 2i)^k$$

**(6 Marks)**

**A5.** Prove the extended Cauchy's integral formula. **(8 Marks)**

**SECTION B: Answer ANY three questions [60].**

**B6.** (a) Evaluate

$$I = \oint_{|z-1|=2} \frac{\sin \frac{\pi z}{2}}{z^2 + 2z - 3} dz$$

**(5 Marks)**

(b) Use the extended Cauchy's integral to evaluate

$$I = \oint_{\left|z-\frac{1}{2}\right|=1} \frac{\cosh z}{(z+1)^3(z-1)} dz$$

**(4 Marks)**

(c) The potential $\phi(\theta)$ between the two infinitely long conducting plates forming an infinite wedge satisfies Laplace's equation in polar coordinates in the form

$$\frac{d^2\phi}{d\theta^2} = 0$$

(i) Solve the differential equation subject to the boundary conditions

$$\phi\left(\frac{\pi}{4}\right) = 30 \quad \text{and} \quad \phi(0) = 0$$

**(4 Marks)**

(ii) Find the complex potential $\Omega(z)$. **(4 Marks)**

(iii) Sketch the equipotential curves and the lines of force. **(3 Marks)**

**B7.** (a) (i) Prove the ML Theorem. **(5 Marks)**

(ii) Use the ML Theorem to bound the function:

$$I = \oint_c \frac{dz}{z(z-2)^3}$$

where $c$ is a circle, centre origin and radius 5. **(5 Marks)**

(b) Find the Laurent series of

$$f(z) = \frac{e^z}{z}, \quad \text{about} \quad z = 0$$

(i) using the formula for $An$ **(5 Marks)**

(ii) using the Taylor series for $e^z$ **(5 Marks)**

**B8.** (a) Define a Pole. **(2 Marks)**

(b) Prove that if $f(z)$ has a simple pole at $z = z_\circ$, then

$$Res(f(z), z_\circ) = \lim_{z \to z_\circ} f(z)$$

**(5 Marks)**

(c) Evaluate $\oint_C \frac{1}{(z-1)^2(z-3)} dz$, where

(i) the contour $C$ is the rectangle defined by $x = 0$, $x = 4$, $y = -1$, $y = 1$, **(4 Marks)**

(ii) and the contour $C$ is the circle $|z| = 2$. **(3 Marks)**

(d) Evaluate (Inverse Laplace Transform)

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{(s-1)(s-3)}\right\}, \quad Re(s) > 3$$

**(6 Marks)**

**B9.** (a) Evaluate

$$I = \int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

**(7 Marks)**

(b) (i) Prove the Jordan's lemma. **(7 Marks)**

(ii) Using the Jordan's lemma, evaluate

$$I = \int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)^2} dx, \quad a \neq 0$$

**(6 Marks)**