GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS II

EMG 1201; EMN 1201

Examination Paper

JUNE 2024

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. M. Mpofu**

INSTRUCTIONS

Candidates should answer **ALL** questions in Section A and attempt **ANY TWO** questions in Section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. (i) Define

(a) Initial-value problem [2]

(b) Partial differential equation [2]

(c) Linear nth-order ordinary differential equation [2]

(ii) Find the general solution

$$xy' = 2y + x^3 e^x$$

[4]

A2. (i) Solve $y'' - 2y' + y = e^x$ [5]

(ii) A mass m is free to move along a plane and is subject to a spring force $-k^2x$, a friction force $-bx''$, and an external force $F(t)$ as indicated in figure 1. For this system, $m = 1$, $b = 4$, $k^2 = 20$, $F(t) = 0$, $x(0) = 5$, and $x'(0) = 10$. Formulate and solve an initial value problem to model the system. Interpret your results. [5]

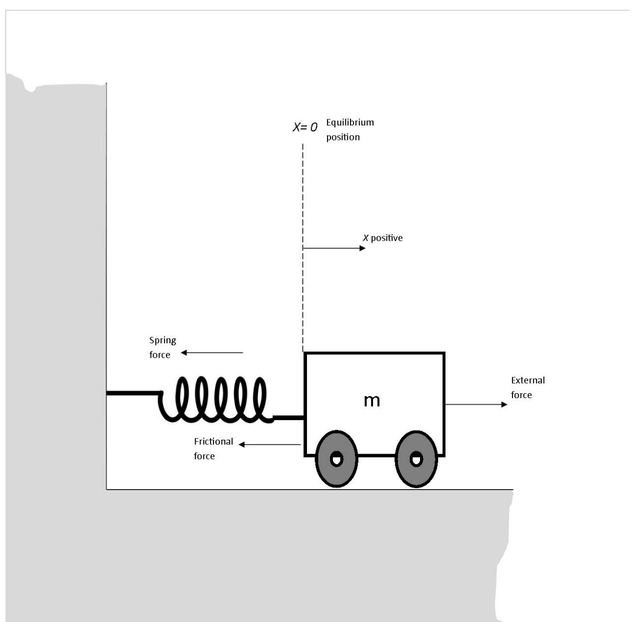

Figure 1:

A3. (i) Find the general solution of the following simultaneous p.d.e.s

$$\frac{\partial u}{\partial x} = 2x + \sin y - ye^{-x}$$

$$\frac{\partial u}{\partial y} = x\cos y + \cos y + e^{-x}$$

[5]

(ii) Find the Fourier whole range series for $f(x) = 2x - 1$ on $0 \leq x \leq 2$. [5]

A4. (i) Evaluate $L^{-1}\left(\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right)$ [5]

(ii) Use the Laplace transform to solve

$$y'' - 4y = 3e^{-t} \qquad y(0) = 1, \quad y'(0) = -1$$

[5]

SECTION B (60 marks) Answer ANY TWO questions from this section.

B5. (i) Define an Explicit solution. [2]

(ii) Solve

(a) $y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$ [4]

(b) $\left(1 + \ln x + \frac{y}{x}\right) dx = (1 + \ln x) dy$ [4]

(c) $y'' + 2y' + y = 0$, $y(0) = 0$, $y'(0) = 3$. [3]

(iii) A mass of 40 g stretches a spring 10 cm. A damping device imparts a resistance to motion numerically equal to 560 times the instantaneous velocity. Find the equation of free motion if the mass is released from the equilibrium position with a downward velocity of 2 cms^{-1}. [5]

(iv) Consider an RLC-circuit modeled by

$$LI'' + RI' + \frac{1}{C} I = E'(t) = E_{\circ}\omega \cos \omega t$$

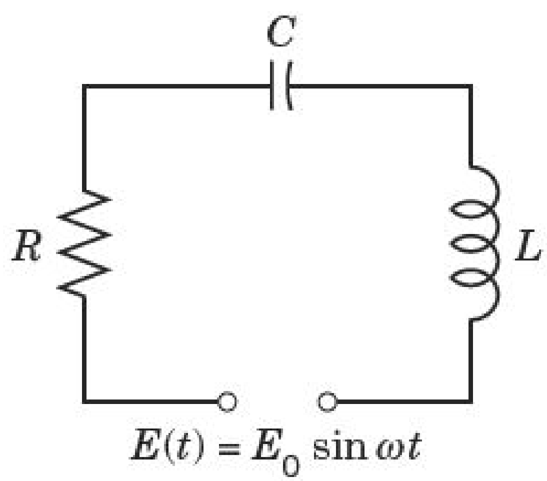

Figure 2: RLC-circuit

(a) Find the current $I(t)$ in an RLC-circuit with $R = 11\ \Omega$ (ohms), $L = 0.1\ H$ (henry), $C = 10^{-2}\ F$ (farad), which is connected to a source of EMF $E(t) = 110 \sin(60 \cdot 2\pi t) = 110 \sin 377t$ (hence 60 Hz = 60 $cycles/sec$). Assume that current and capacitor charge are 0 when $t = 0$. [10]

(b) Sketch the graph of current I against time t. [2]

B6. (i) Show that $u(x, y) = y^2 - x^2$ and $u(x, y) = e^y \sin x$ are solutions to Laplace's equation $u_{xx} + u_{yy} = 0$. [3]

(ii) Use the seperation of variables to solve

(a) $x \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ [5]

(b) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ given
$u(0, y) = u(10, y) = 0$
$u(x, 0) = 5\sin(\frac{\pi x}{2}) - 3\sin(\frac{3\pi x}{2})$
$\frac{\partial u(x,0)}{\partial y} = 0$ [8]

(iii) Consider a long thin copper bar of constant cross section and homogenous material, which is oriented along the x-axis as shown in Figure 3 and is perfectly insulated laterally, so that the heat flows in the x-direction only and is modeled by a one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $c^2 = K/(\sigma\rho)$, $u(x, 0) = f(x)$ is the initial condition and the boundary conditions are given by $u(0, t) = 0$, $u(L, t) = 0$, $for\ all\ t \geq 0$.

Figure 3: Copper bar

(a) Show that the general solution of the heat equation is given by

$$u(x, t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L} e^{\lambda^2 t} \qquad (n = 1, 2, \cdots)$$

. where $\lambda = \frac{cn\pi}{L}$. State initial conditions in-terms of sin, B_n, n, π and x. [8]

(b) Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin(\pi x/80)$ $°C$ and the ends are kept at $0°C$. How long will it take for the maximum temperature in the bar to drop to $50°C$? (Physical data for copper: density ρ 8.92 gcm^{-3}, specific heat σ 0.092 $cal/(g°C)$, thermal conductivity K 0.95 $cal/(cms°C)$) [6]

B7. (i) Find the fourier series of

(a) $f(x) = 1 - |x|$, $-3 \leq x \leq 3$, $f(x+6) = f(x)$ [5]

(b) $f(x) = x$, $0 < x < 1$ [6]

(ii) Find the transform of

$$f(x) = H(x-2)e^{-kx}$$

[6]

(iii) A rectifier is an electrical component that converts alternating current (AC) to direct current (DC). Half-wave rectifier can be used as a voltage multiplier. A sinusoidal voltage $M\sin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave.
Find the Fourier series of the resulting periodic function

$$y(t) = \begin{cases} 0 & -L < t < 0, \\ M\sin\omega t & 0 < t < L \end{cases}$$

where $p = 2L = \frac{2\pi}{\omega}$, $L = \frac{\pi}{\omega}$ [13]

END OF QUESTION PAPER