GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS II

EMG 1201; EMN 1201

Examination Paper

April 2025

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. M. Mpofu**

INSTRUCTIONS

Candidates should answer **ALL** questions in Section A and attempt **ANY TWO** questions in Section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. (i) Distinguish between Ordinary differential equation and Partial differential equations [2]

(ii) Solve,

(a) $\dfrac{dy}{dx} = e^{-x^2}$, $y(3) = 5$ [4]

(b) $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} - 5y = 0$, $y(1) = 0, y'(1) = 2$ [4]

A2. (i) Solve of the Initial-Value Problem (IVP)

$$\mathbf{X}' = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix} \mathbf{X}, \quad \mathbf{X}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

[7]

(ii) Hence, sketch the phase portrait of the IVP. [2]

A3. (i) Find the fourier series of

$$f(x) = \begin{cases} x, & -\pi < x \leq 0 \\ -x, & 0 < x \leq \pi \end{cases} \quad \text{and} \quad f(x + 2\pi) = f(x)$$

[5]

(ii) Solve the boundary value problem

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \ , \quad \text{given} \quad u(y, 0) = 4e^{-y} - e^{-5y}$$

[5]

A4. (i) Evaluate $\mathcal{L}^{-1}\left[\frac{2s-1}{(s^2+s)(s-1)}\right]$. [5]

(ii) Solve the IVP $y'' - 3y' - 2y = e^{-4t}$, $y(0) = 1$, $y'(0) = 5$ using the Laplace transforms. [6]

SECTION B (60 marks) Answer ANY TWO questions from this section.

B5. (i) Define Homogeneous linear nth-order ordinary differential equation [2]

(ii) Solve

(a) $L\dfrac{di}{dt} + Ri = E$, $i(0) = i_\circ$, L, R, E, and $i_\circ$ constants. [4]

(b) $\left(\dfrac{3y^2 - t^2}{y^5}\right)\dfrac{dy}{dt} + \dfrac{t}{2y^4} = 0$, $y(1) = 1$ [4]

(iii) Solve

$$\frac{dx}{dt} = -4x + y + z$$
$$\frac{dy}{dt} = x + 5y - z$$
$$\frac{dz}{dt} = y - 3z$$

[8]

(iv) Find the motion of the mass-spring system modeled by the ODE and the initial conditions.

$$\left(D^2 + 2D + 2I\right) y = e^{-t/2} \sin \frac{1}{2}t, \quad y(0) = 0, \quad y'(0) = 1$$

Sketch the solution of the curve. [9, 3]

B6. (i) Define a general linear first order p.d.e. [2]

(ii) Write the following differential equations in Sturm-Liouville form:

(a) $y'' - xy' + \lambda y = 0$ [3]

(b) $xy'' + (1 - x)y' + \lambda y = 0$ [3]

(iii) Find the eigenvalues and eigenfunctions for the S-L problem

$$y'' - y' + \lambda y = 0, \quad y(0) = y(\pi) = 0$$

[5]

(iv) Solve

(a) the simultaneous p.d.e:

$$\frac{\partial u}{\partial x} = xy \ \text{ and } \ \frac{\partial u}{\partial y} = 2y + x$$

[5]

(b) by seperation of variables

$$\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0; \quad u(x, 0) = 3e^{-5x}$$

[5]

(v) Consider a long thin copper bar of constant cross section and homogenous material, which is oriented along the x-axis as shown in Figure 1 and is perfectly insulated laterally, so that the heat flows in the x-direction only and is modeled by a one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $c^2 = K/(\sigma\rho)$, $u(x,0) = f(x)$ is the initial condition and the boundary conditions are given by $u(0,t) = 0, \quad u(L,t) = 0, \quad for\ all\ t \geq 0.$

Figure 1: Copper bar

Show that the general solution of the heat equation is given by

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L} e^{\lambda^2 t} \qquad (n = 1, 2, \cdots)$$

where $\lambda = \frac{cn\pi}{L}$. State initial conditions in-terms of sin, B_n, n, π and x. [8]

B7. (i) Define unit step function. [2]

(ii) Find the Fourier transform of

(a) $f(x) = H(x)e^{-kx}$ [6]

(b) $f(x) = \begin{cases} 0 & x \leq 0 \\ e^{-kx}\cos(2x) & x > 0,\ k > 0 \end{cases}$ [6]

(iii) (a) Find the fourier series of

$$V(x) = \begin{cases} 1 & -\pi < x \leq 0 \\ -1 & 0 < x \leq \pi \end{cases} \quad \text{and} \quad V(x + 2\pi) = V(x)$$

[6]

(b) Hence, solve

$$\frac{d^2 y}{dx^2} + 10y = V(x)$$

[10]

END OF QUESTION PAPER