GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENTS OF MATHEMATICS AND STATISTICS

CALCULUS I

CMS 1101

Examination Paper

NOVEMBER 2024

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. M. Mpofu**

<u>INSTRUCTIONS</u>

Candidates should answer **ALL** questions in Section A and attempt **ANY THREE** questions in Section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)
Answer ALL questions from this section.

A1. (i) Solve the inequality

(a) $x^3 + 3x < 4x^2$ [3]

(b) $|5x - 2| \leq 6$ [3]

(ii) Find the Maclaurin series of $f(x) = \ln(1-x)$ [4]

A2. (i) Define continuity. [2]

(ii) Given that

$$F(x) = \frac{1}{x^2 - x}$$

(a) Find the domain $F(x)$ [2]

(b) If $F = f \circ g$, find f and g [3]

(iii) Prove that

$$\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$$

[3]

A3. (i) Verify the identity

$$f(x)g''(x) - f''(x)g(x) = \frac{d}{dx}\left[f(x)g'(x) - f'(x)g(x)\right]$$

[4]

(ii) Newton's Law of Gravitational Attraction states that if two bodies are at a distance r apart, then the force F exerted by one body on the other is given by

$$F(r) = \frac{k}{r^2}$$

where k is a positive constant. Suppose that, as a function of time, the distance between the two bodies is given by

$$r(t) = 49t - 4.9t^2, \quad 0 \leq t \leq 10.$$

(a) Find the rate of change of F with respect to t. [3]

(b) Show that $(F \circ r)'(3) = -(F \circ r)'(7)$. [3]

A4. (i) Show that $\sum_{n=0}^{\infty} x^n/n!$ converges for all x. [5]

(ii) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid. [5]

SECTION B (60 marks)
Answer ANY THREE questions from this section.

B5. (i) Define a complex-valued function. [2]

(ii) Given $z = 1 + i$ and $w = \sqrt{3} - i$

(a) Find the polar form of z and w. [4]

(b) Find the product of z and w in polar form. [4]

(iii) Evaluate

$$\lim_{x \to 9} \frac{x - 9}{\sqrt{x + 7} - 4}$$

[4]

(iv) Assume that

$$f(x) = \begin{cases} 2 + \sqrt{x} & \text{if } x \geq 1 \\ \frac{x}{2} + \frac{5}{2} & \text{if } x < 1. \end{cases}$$

(a) Determine whether or not f is continuous at $x = 1$. Justify your answer and state your conclusion. [2]

(b) Using the definition of the derivative, determine $f'(1)$. [4]

B6. (i) Find the first derivative of the function

$$y = \cos\left((8t + 9)^{-4/7}\right)$$

[4]

(ii) Consider the curve $y = f(x)$ for the function

$$f(x) = \frac{(2 + x)^2}{1 + x}$$

(a) Identify the domain of f and any symmetries the curve may have.

(b) Find $f'(x)$ and $f''(x)$.

(c) Find the critical points of f, and identify the function's behaviour at each one.

(d) Find where the curve is increasing and where it is decreasing.

(e) Find the points of inflection, if any occur, and determine the concavity of the curve.

(f) Identify any asymptotes.

(g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve. [16]

B7. (i) Find the Taylor series generated by $f(x) = \sin\left(2x + \frac{\pi}{2}\right)$ at $x = \frac{\pi}{4}$ [5]

(ii) (a) Describe an **absolutely convergent series**. [2]

(b) Use the ratio/root test to determine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x + 2)^n}{n5^n}$$

State the radius and interval of convergence of the series. [7]

(iii) Show that $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ diverges. [Hint: $2^{n^2} = (2^n)^n$] [6]

B8. (i) Evaluate

(a) $\int_{-\pi}^{0} \sin(6x)\cos(3x)dx$ [5]

(b) $\int x^3 e^{x^4} dx$ [5]

(ii) Consider

$$\int_2^4 \frac{1}{(s-1)^2} ds$$

(a) Use the Simpson's rule to estimate the integral with $n = 4$ steps. [4]

(b) Determine the accuracy of the technique. [2]

(iii) Find the length of curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \leq x \leq 3$$

[4]

END OF QUESTION PAPER