GWANDA STATE UNIVERSITY

CMS2105

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

ORDINARY DIFFERENTIAL EQUATIONS

EPOCH MINE CAMPUS

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SEPTEMBER 2024: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY TWO** questions from **Section B** (30 marks each).

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

A1. (a.) Show that e^x is a solution of:

$$xy^{''} - (1+x)y^{'} + y = 0$$

(b.) Hence find the general solution of:

$$xy^{''} - (1+x)y^{'} + y = x^2e^{2x}.$$

[2,10]

A2. (a) Solve the I.V.P:
$cos(x)y^{'} + sin(x)y = 2cos^3(x)Sin(x) - 1, y(\pi/4) = 3\sqrt{2},\ 0 \leq x \leq \pi/2$ [8]

(b.) Solve $\dfrac{dy}{dx} = 6y^2x,\ y(1) = (1/25).$ [6]

A3. Solve $\dfrac{d^2x}{dt^2} = -4\dfrac{dx}{dt} - 3x$ [8]

A4. Determine the Taylor series for $f(x) = e^x$ about $x = 0$. [6]

SECTION B: Answer ANY TWO questions [60].

B5. (a.) Solve the differential equation

$$(ycosx + 2xe^y) + (sinx + x^2e^y - 1)\frac{dy}{dx} = 0 \quad [8]$$

(b.) Let $y_1(x)$ and $y_2(x)$ be any two distinct solutions of the second order differential equation;

$a_2(x)y^{''} + a_1(x)y^{'} + a_0(x)y = 0, x \in I$

(i) Show that;

$$\frac{d}{dx}\left[\frac{y_2(x)}{y_1(x)}\right] = \frac{W[y_1(x), y_2(x)]}{[y_1(x)]^2}$$

. [3]

(ii) Use (a) above to show that if $W[y_1(x), y_2(x)] = 0$ in I, then the set $y_1(x), y_2(x)$ is not a fundamental set (i.e not linearly independent) on I [5]

(iii) Show that every separable first order differential equation is exact. [4]

(c.) Find the series solution of $y^{''} = xy$ and the point $x = 1$. [10]

B6. (a.) Let $Lf(t) = F(s)$ define a Laplace transform of the function $f(t)$. Prove the following;

(i) $L\ f^{''}(t) = s^2F(s) - sf(0) - f^{'}(0)$;

(ii) $L\ t^2f(t) = (-1)^2F^{''}(s)$;

(iii) Solve the equation:

$$y^{''} - 3y^{'} + 2y = e^{3t}, y(0) = 1, y^{'}(0) = 0$$

[5, 5,10]

(b.) Consider the equation:

$$\phi(t) + \int_t^0 k(t-x)\phi(x)dx = f(t)$$

in which f and k are known functions and ϕ is to be determined. Taking the Laplace transforms of the given equation, obtain an expression for $L\phi(t)$ in terms of the transforms $Lf(t)$ and $Lk(t)$. i.e show that

$$\phi(s) = \frac{F(s)}{1 + K(s)}$$

[10]

B7. (a.) Solve $y'' - y' - 2y = e^{3x}$. [10]

(b.) Prove that if $\phi(x)$ and $\psi(x)$ are any two solutions of the non- homogeneous linear differential equation;

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = f, a_0(x) \neq 0$$

on some interval I, then $\phi - \psi$ is a solution of the corresponding homogeneous equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ on I. [5]

(c.) Verify that $y = x^{-1}$ is a solution of

$$2x^2y'' + 3xy' - y = 0, x > 0.$$

Hence find the general solution. [10]

d.) Solve $\dfrac{dy}{dx} = \dfrac{x^2 + 2}{y}$. [5]