GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

NUMERICAL METHODS AND ORDINARY DIFFERENTIAL EQUATIONS

ESG 2212

Examination Paper

APRIL 2024

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. R. G. Moyo**

INSTRUCTIONS

Candidates should answer **ALL** questions in section A and **ANY THREE** questions in section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Define the following terms

(i) Round-off error [2]

(ii) Augmented matrix [2]

(iii) LU decomposition [2]

A2. (i) Outline the steps followed when solving a non-linear equation using the bisection method [4]

(ii) Use the Bisection Method to find the root of $f(x) = \cos x - \sqrt{x}$ lying between [0,1]. Use a tolerance of $\epsilon = 0.01$ and give your answer correct to 5 decimal places [7]

A3. (i) Use the Simpson's rule with $n = 10$ to obtain an approximation to $\int_0^2 xe^{3x}dx$ correct to 5 decimal places. [6]

(ii) Consider the following table

x	1.3	1.4	1.45	1.5	1.55	1.6
$f(x)$	4.77	5.677	6.182	6.732	7.3028	7.9248

Evaluate

(a) $f'(1.4)$ using the forward difference method [3]

(b) $f'(1.5)$ using the central difference method for $h = 0.1$ [3]

(c) $f''(1.6)$ [4]

A4. Use the following values to construct a third degree Lagrange approximation to $f(1.25)$ correct to 5 decimal places : [7]

$f(1.0) = 1.00000$
$f(1.1) = 1.23368$
$f(1.2) = 1.55271$
$f(1.3) = 1.99372$

SECTION B (60 marks)

Answer ANY THREE questions from this section.

B5. (i) Determine the local truncation error (LTE) when solving an initial value problem using the Taylor's method. [3]

(ii) Use Taylor's fourth order to solve $y' = 2x - y$, $y(0) = 1$ over $[0, 2]$ using $h = 0.5$ [17]

B6. (i) Discuss the application of Error analysis in Geomatics and Surveying. [5]

(ii) Solve the following system of equations using the Gauss Jordan elimination [15]

$$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$$
$$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$$
$$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$$
$$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$$

B7. (i) Construct a linear interpolation function for the ponts $(3, 2)$ and $(5, 8)$ [3]

(ii) Differentiate between precision and accuracy [6]

(iii) Assume that we have developed instrumentation to measure the velocity of the parachutist. The measured data obtained for a particular test case are
Use Newton's interpolation formula $P_4(x)$ to estimate the velocity of the parachuist

Time (s)	1	3	5	7	13
Velocity (cm/s)	800	2310	3090	3940	4755

when the time is 10 seconds [11]

B8. (i) Write a Matlab code to solve an Initial value problem $y' = \frac{x-y}{2}$, $y(0) = 1$ using the Runge-Kutta Method of order 4. [8]

(ii) Solve the following system of equations using the LU decomposition method. [12]
$$6x + 18y + 3z = 3$$
$$2x + 12y + z = 19$$
$$4x + 15y + 3z = 0$$

END OF QUESTION PAPER