GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF MINING AND METALLURGY

ENGINEERING MATHEMATICS I/CALCULUS

EMG 1101; EMN 1101; ESG 1207

Semester One 2026 : Examination

This examination paper consists of 4 printed pages, exclusive of this cover page.

Date:	**JUNE 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Mr. M. Mpofu**

INSTRUCTIONS

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY THREE** questions from **Section B** each question carries **20 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

ADDITIONAL MATERIALS

- Calculator (non-programmable)
- Pens and Pencils

SECTION A: Answer ALL questions [40].

A1. Solve

(a) $|x+3| = |2x+1|$ **(3 Marks)**

(b) $x^2 + 7x - 8 < 0$ **(4 Marks)**

A2. (a) Given

$$f(x) = \frac{e^x}{1+2e^x},$$

find f^{-1}. **(4 Marks)**

(b) Evaluate $\lim_{x\to\infty}(x^2 - x)$ **(3 Marks)**

(c) Show that

$$\int_a^b \left(\int_x^b f(t)dt\right) dx = \int_a^b (x-a)f(x)dx$$

(5 Marks)

A3. (a) Compute $\frac{d^2}{dx^2}\left[(x^2-3x)\frac{d}{dx}(x+x^{-1})\right]$ **(4 Marks)**

(b) Evaluate

$$\int_0^{\pi} e^x \cos x dx$$

(4 Marks)

(c) A vertical mine shaft 100 meters deep is filled with water. The cross-sectional area of the shaft is 10 m^2. The density of water is 1000 kg/m^3.
Calculate the total work required to pump all the water to the surface. **(4 Marks)**

A4. (a) Let $S_n = a + ar + \cdots + ar^{n-1}$, with $r \neq 1$. Show that $S_n = \frac{a(r^n-1)}{r-1}$ **(4 Marks)**

(b) Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=0}^{\infty}(-1)^n \frac{(x-3)^n}{2n+1}$$

(5 Marks)

SECTION B: Answer ANY three questions [60].

B5. (a) Define a continuous function. **(2 Marks)**

(b) Consider $f(x) = 2x^2 - 3x + 1$ and $g(x) = x^2 + 1/x$, compute

$$\left(\frac{f+2g}{f}\right)(-1)$$

(3 Marks)

(c) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

(3 Marks)

(d) Evaluate $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$ **(4 Marks)**

(e) Find the six sixth roots of $z = -8$ and graph these roots in the complex plane **(8 Marks)**

B6. (a) The heat equation is a partial differential equation, which describes the distribution of heat in a region at time t can be represented by the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Show that $u(x,t) = \sin(\alpha x) \cdot e^{-\beta t}$ satisfies the heat equation for constants α and β. What is the relationship between α and β for this function to be a solution? **(4 Marks)**

(b) Water flows into a conical tank (see illustration) at a rate of 0.6 *cubic metres per minute*. How fast is the water level h rising when the water is 2.3 *metre* deep? (Volume of a cone is $\frac{1}{3}\pi r^2 h$.) **(4 Marks)**

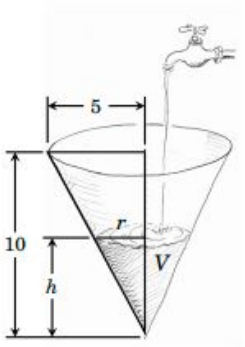


Figure 1:

(c) Sketch the graph

$$f(x) = \frac{(x+2)^2}{x^2+2}$$

Determine all relative extrema, inflection points, asymptotes, concavity and suggest the behaviour at infinity. **(12 Marks)**

B7. (a) Define integrable function. **(2 Marks)**

(b) Evaluate

$$\int x^2 \sin 3x dx$$

(5 Marks)

(c) Consider the integral

$$\int_0^1 e^{x^2} dx$$

For $n = 5$, approximate the value of the integral using

(i) the Trapezium rule **(3 Marks)**

(ii) the Simpson's rule **(4 Marks)**

(d) Answer the following questions

(i) An engineer is analyzing the velocity $v(t)$ of a piston. If the integral $\int_0^T v(t)dt = 0$ over one full cycle T, what does this result represent physically? **(2 Marks)**

(ii) In an optimization problem for minimizing material for a cylindrical container, why is it necessary to check the boundaries of the domain? **(2 Marks)**

(iii) If $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$, what does this imply about the relationship between differentiation and integration? **(2 Marks)**

B8. (a) Find the Taylor series generated by

$$f(x) = \sin\left(3x + \frac{\pi}{2}\right)$$

at $x = \pi/4$. **(5 Marks)**

(b) Using the ratio test or root test, determine if the series converges absolutely or diverges

$$\sum_{n=1}^{\infty} (-1)^n \left(1 - \frac{1}{n}\right)^{n^2}$$

(5 Marks)

(c) A cylindrical riser for a casting needs to be designed to minimize surface area (to reduce heat loss) while maintaining a necessary volume (V) of 2000 cm^3 to prevent shrinkage defects. Find the radius (r) and height (h) that minimize the surface area $S = 2\pi r^2 + 2\pi rh$ given the constraint $\pi r^2 h = 2000$ **(5 Marks)**

(d) Calculate

$$\int\int_R 100 - 6x^2 y dA, \qquad R: \ 0 \leq x \leq 2, \quad -1 \leq y \leq 1$$

(5 Marks)