GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

NUMERICAL METHODS AND ORDINARY DIFFERENTIAL EQUATIONS

ESG 2212

Semester Two 2025/2026 : Examination Paper

This examination paper consists of 2 printed pages, cover page not included.

Date:	**JUNE 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Mr. R.G. Moyo**

INSTRUCTIONS

- Candidates should attempt **ALL** questions from **Section A** and **ANY THREE** questions from **Section B**. Section A carries 40 marks and Section B carries 60 marks.
- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.

ADDITIONAL MATERIALS

- Scientific Calculator (non-programmable)

SECTION A: Answer ALL questions [40 marks].

A1. Define the following terms

(i) Truncation error [2]

(ii) Numerical Ill-conditioning [2]

(iii) Runge-Kutta Method of Order Four (RK4) [2]

A2. Verify that when the Newton-Raphson method is used to compute $\sqrt{D}$, then the sequence of iterates is defined by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{D}{x_n}\right)$$

Hence use an iterative process to evaluate $(70)^{\frac{1}{2}}$ correct to 6 decimal places starting with $x_0 = 7.237$. Give your answer correct to 6 decimal places [9]

A3. (i) Use Simpson's Rule to integrate the points represented by the table below

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.669	1.811	1.971	2.151	2.325	2.577	2.828	3.107

(a) using $h = 0.1$ [4]

(b) using $h = 0.2$ [4]

(ii) Consider the following table

x	1.3	1.4	1.45	1.5	1.55	1.6
$f(x)$	4.77	5.677	6.182	6.732	7.3028	7.9248

Evaluate

(a) $f'(1.5)$ using the forward difference method for $h = 0.05$ [3]

(b) $f'(1.5)$ using the central difference method for $h = 0.1$ [3]

(c) $f''(1.4)$ [4]

A4. Use the following values to construct a third degree Lagrange approximation to $f(1.25)$ correct to 5 decimal places :
$f(1.0) = 1.00000$
$f(1.1) = 1.23368$
$f(1.2) = 1.55271$
$f(1.3) = 1.99372$ [7]

SECTION B: Answer ANY three questions [60 marks].

B5. (i) State the three elementary row equations used when solving a system of linear equations. [3]

(ii) Given the following system of equations

$$\begin{aligned} 6x_1 - 2x_2 + 2x_3 + 4x_4 &= 16 \\ 12x_1 - 8x_2 + 6x_3 + 10x_4 &= 26 \\ 3x_1 - 13x_2 + 9x_3 + 3x_4 &= -19 \\ -6x_1 + 4x_2 + x_3 - 18x_4 &= -34 \end{aligned}$$

(a) Write down the system in the form $\mathbf{A}\overline{x} = b$ [3]

(b) Find the inverse of the matrix $\mathbf{A}$ using elementary row operations [10]

(c) Hence use the inverse of matrix $\mathbf{A}$ to solve the system [4]

B6. Given an initial value problem (IVP) defined by $y' = 2xy + y^2$, $y(1) = 1$

(i) Evaluate $y(1.3)$ using the Taylor's third order method. Use $h = 0.1$ [12]

(ii) Compute $y(1.2)$ using Runge-Kutta Method of order 4 (RK4) using $h = 0.1$ [8]

B7. (i) Differentiate between Interpolation and Extrapolation [2]

(ii) Construct a linear interpolation function for the ponts (3, 2) and (5, 8) [3]

(iii) The following data were obtained from measurements taken during a surveying experiment:

x	1	2.5	3	5	9
$f(x)$	2.0	2.8	3.6	5.1	6.3

(a) Construct the Newton divided difference interpolation polynomial for the data. [6]

(b) Use the polynomial to estimate the value of $f(4.371)$. [4]

(iv) Discuss the applicability of interpolation in Geomatics and Surveying [5]

B8. (i) Starting with the initial guess $\left(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)}\right) = (0, 0, 0, 0)$, perform three Jacobi iterations for the following system of equations.

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6, \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11, \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$ [10]

(ii) Solve the following system of equations using the Naive-Gaussian Elimination.

$$\begin{aligned} x_1 - 4x_2 + 4x_3 + 7x_4 &= 4 \\ 2x_2 - x_3 &= 5 \\ 2x_1 + x_2 + 2x_3 + 4x_4 &= 2 \\ 2x_1 - 3x_2 + 2x_3 - 5x_4 &= 9 \end{aligned}$$ [10]

END OF QUESTION PAPER