GWANDA STATE UNIVERSITY

SMS1201

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

CALCULUS II

EPOCH MINE CAMPUS

Ms B Kwirira

JUNE 2024: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY TWO** questions from **Section B** (30 marks each).

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

A1. (a) Define what is **meant** by saying two vectors **A** and **B** are orthogonal. [2]

(b) Let $\mathbf{A} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j}$. Find a unit vector orthogonal to both **A** and **B**. [6]

A2. The epicycloid C is given by $\mathbf{r}(t) = (5\cos t - \cos 5t)\mathbf{i} + (5\sin t - \sin 5t)\mathbf{j},\ 0 \leq t \leq 2\pi$. Find the intervals in which C is smooth. [6]

A3. Determine the nature of stationery points for the function

$$f(x, y) = x^2 - 2xy + 2y^2 - 2x + 2y + 4.$$

[6]

A4. Let $f(x, y, z) = x^2 \ln(xyz)\sin(yz)$. Find the gradient of the function $f(x, y, z)$. [8]

A5. Evaluate $\displaystyle\int_1^4 \int_0^1 \int_0^x 2ze^{-x^2}\, dydxdz.$ [6]

A6. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{3^n}$ converges or diverges. [6]

SECTION B: Answer ANY two questions [60].

B7. (a) Find a set of parametric equations of the line that passes through the points $P(-2,1,0)$ and $Q(1,3,5)$. [6]

(b) Let $A=(1,1,1)$, $B=(0,3,1)$ and $C=(-1,2,4)$ be points on the plane P_1. Find vectors $\mathbf{AB}$ and $\mathbf{AC}$. Hence or otherwise, find the equation of the plane P_1. [10]

(c) Let P_2 be a plane with equation $2x+3y+4z=5$. Find the line of intersection between P_1 in question $B7(b)$ and the plane P_2. [6]

(d) A particle moves along the curve C given by $\mathbf{r}(t) = 2\sin\frac{t}{2}\mathbf{i} + 2\cos\frac{t}{2}\mathbf{j}$. Find in terms of t the velocity vector, acceleration vector and the speed of the particle. [3,3,2]

B8. (a) (i) Define what is **meant** by saying the function $f(x,y)$ converges to l as (x,y) tends to a point (x_0,y_0). [2]

(ii) Use the definition to show that $\lim_{(x,y)\to(2,3)} x^2+2y=10$. [8]

(b) Show that the function

$$f(x,y) = \begin{cases} \left(\dfrac{x^2-y^2}{x^2+y^2}\right)^2, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

is not continuous at the point $(0,0)$. [4]

(c) Let $u=x+y$ and $v=xy$. Find $\dfrac{\partial x}{\partial u}$, $\dfrac{\partial x}{\partial v}$, $\dfrac{\partial y}{\partial u}$ and $\dfrac{\partial y}{\partial v}$. [8]

(d) Find the equation of the tangent plane to the hyperboloid given by

$$z^2 = 2x^2+2y^2+12$$

at the point $(1,-1,4)$. [8]

B9. (a) The spherical coordinates are given by $x=r\cos\theta\sin\phi$, $y=r\sin\theta\sin\phi$ and $z=r\cos\phi$. Show that the Jacobian from (x,y,z) to (r,θ,ϕ) is given by $-r^2\sin\phi$. [10]

(b) Sketch the region of integration R of the double integral $\int\int_R (2x^2y^{-2}+2y)\,dydx$, where R is bounded by $1\le x\le 2$ and $1\le y\le x$. Hence or otherwise evaluate $\int\int_R (2x^2y^{-2}+2y)\,dydx$. [10]

(c) Find $\int\int_R (x^2+y^2)\,dxdy$, where R is the region bounded by $xy=1$, $xy=3$, $x^2-y^2=1$ and $x^2-y^2=4$. [10]