GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS IV

EMN/EMG 2201

Semester Two 2025/2026 : Examination Paper

This examination paper consists of 4 printed pages, cover page not included.

Date:	**JUNE 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Mr. R.G. Moyo & Mr G. Maswera**

INSTRUCTIONS

- Candidates should attempt **ALL** questions from **Section A** and **ANY THREE** questions from **Section B**. Section A carries 40 marks and Section B carries 60 marks.
- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.

ADDITIONAL MATERIALS

- Scientific Calculator (non-programmable)
- Statistical Tables

SECTION A: Answer ALL questions [40 marks].

A1. Define the following terms as they are used in Statistics

(i) Latin square [2]

(ii) Hypothesis Test [2]

(iii) Replication [2]

A2. A research institute recruits interns from three colleges: College A, College B, and College C. Of all recruits, 50% come from College A, 30% from College B, and 20% from College C. Among interns from College A, 4% fail to submit reports on time. The corresponding probabilities for College B and College C are 10% and 15% respectively.

(i) Draw a probability tree diagram to illustrate the situation. [2]

(ii) Find the probability that the next intern recruited

(a) will be from College B and will submit the report on time. [3]

(b) will submit the report on time. [4]

A3. A mining company states that its ore crushing process produces fragments with an average size of 500 micrometres. A random sample of 36 crushed ore batches is taken and the mean fragment size is found to be 492 micrometres. Assume the population standard deviation is 24 micrometres.

Test, at the 5% level of significance, whether the true mean fragment size differs from the stated value. [7]

A4. If X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{4}, & 0 \leq x \leq 2, \\ -\frac{x}{4} + 1, & 2 \leq x \leq 4, \\ 0, & \text{otherwise}, \end{cases}$$

(i) Sketch $f(x)$. [2]

(ii) Find $P(1 \leq x \leq 3.5)$. [3]

(iii) Calculate $\text{Var}(x)$. [4]

A5. The Department of Mining Engineering has a total of 120 male and 80 female students. A random sample of 12 students is selected from this department. Find

(i) the probability that exactly 5 students in the sample are female. [3]

(ii) the expected number of male students in the sample. [2]

(iii) the variance of the number of female students in the sample. [4]

SECTION B: Answer ANY three questions [60 marks].

B6. In an attempt to deepen their understanding of phase transformation kinetics and the practical implications of heat treatment on ferrous alloys, a group of final-year Metallurgical Engineering students conducted a rather elaborate laboratory investigation on medium-carbon steel specimens. The primary objective of the experiment was to examine how the combined influence of quenching severity and subsequent tempering temperature affects the resulting hardness of the material.

During the experimental programme, three distinct quenching media were considered, each expected to produce different cooling rates and hence different martensitic structures:
Water: rapid quench in agitated water
Oil: moderate quench in mineral oil
Air: slow cooling in still air.

Following quenching, the specimens were tempered at three different temperature regimes in order to relieve internal stresses and modify the hardnesstoughness balance:
Low: 200°C
Medium: 400°C
High: 600°C.

The experiment was repliated 6 times. The resulting hardness measurements (Rockwell C scale) were obtained in the Metallurgical Engineering laboratory and are summarized below (values are averages of replicate readings taken under controlled conditions):

QUENCHING MEDIUM	TEMPERING **(Low)**	TEMPERING **(Medium)**	TEMPERING **(High)**
Water	62 64 61 63 65 62	55 57 54 56 58 53	48 50 47 49 46 51
Oil	58 60 57 59 61 56	52 50 54 51 53 49	44 46 45 43 47 42
Air	50 52 49 51 53 48	45 47 44 46 43 48	38 40 39 37 41 36

Assuming the usual two-factor fixed-effects model with interaction, perform a complete two-way ANOVA at the 5% level of significance to:

(i) Test whether the quenching medium has a significant effect on hardness.

(ii) Test whether the tempering temperature has a significant effect on hardness.

(iii) Test whether there is a significant interaction between quenching medium and tempering temperature.

(iv) Conduct further comparisons to determine whether significant differences exist among the nine treatment combinations.

Present the full ANOVA table and state your conclusions in clear terms. [20]

B7. (i) Fit a multiple linear regression equation with Productivity as the dependent variable and Operator Experience and Drill Bit Hardness as independent variables. [12]

Operation	1	2	3	4	5	6	7	8
Productivity (m^3/hr)	12.5	14.1	15.0	13.7	16.2	14.8	15.5	16.8
Operator Experience (years)	2	3	5	4	6	3	5	7
Drill Bit Hardness (HRc)	60	62	65	63	68	66	64	69

(ii) The thickness of rolled aluminium sheets produced in a plant is normally distributed with mean 2.5 mm and standard deviation 0.15 mm.

(a) What proportion of sheets have thickness greater than 2.7 mm? [2]

(b) What thickness corresponds to the top 10% of the sheets? [3]

(iii) A sample of 36 steel rods produced by a rolling process has a mean diameter of 15.2 mm. The population standard deviation is known to be 0.6 mm. Construct a 95% confidence interval for the true mean diameter of the rods. [3]

B8. (i) A metallurgical laboratory tests whether the type of flux used in smelting affects slag formation. Slag formation is categorized as Low, Medium, or High. The results from three types of flux are recorded below:

Flux Type	**Low**	**Medium**	**High**
Flux A	12	18	10
Flux B	15	20	15
Flux C	18	15	12

At the 5% significance level, test whether slag formation is independent of flux type. [8]

(ii) In semiconductor manufacturing, wet chemical etching is commonly employed to remove silicon from the backs of wafers prior to metallization. One critical parameter of this process is the etch rate, typically measured in mils per minute. A process engineer wishes to compare the performance of two different etching solutions.

Eight wafers were randomly selected and etched in each solution. The observed etch rates (in mils/min) are as follows:

Solution 1	9.9	10.6	9.4	10.3	10.0	9.3	10.7	10.3
Solution 2	10.6	10.2	10.0	10.7	10.4	10.5	10.3	10.3

Do the data indicate, at the 5% significance level, that the mean etch rates of the two solutions are the same? Assume equal variances. [12]

B9. In a mineral processing laboratory, a group of Mining Engineering students sought to investigate how different flotation reagents influence the recovery efficiency of a finely disseminated copper ore. Previous plant data suggested that reagent chemistry plays an important role in flotation performance. However, the students were also aware that ore samples obtained from different sections of the orebody may exhibit natural variability in mineral composition, liberation characteristics, and surface chemistry.

To control for this nuisance variability, the students designed the experiment using a Randomized Complete Block Design (RCBD), treating ore batches as blocks and flotation reagents as treatments.

Four flotation reagents were selected for evaluation. Five ore batches were obtained from different locations within the pit, and each reagent was applied once to each batch in random order. The response variable measured was copper recovery (%) obtained from standardized laboratory flotation tests.

The experimental results are shown below.

Reagent	**Batch 1**	**Batch 2**	**Batch 3**	**Batch 4**	**Batch 5**
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

Using $\alpha = 0.05$, perform a check on whether the flotation reagent has a significant effect on copper recovery using an appropriate randomized block analysis. State all necessary hypotheses and draw practical mining conclusions. [20]

END OF QUESTION PAPER