



GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENTS OF MATHEMATICS AND STATISTICS

MATHEMATICAL SOFTWARES

CMS 2106

Examination Paper

NOVEMBER 2024

Time Allowed: 3 hours
Total Marks: 100
Examiner's Name: Mr. M. Mpofu

INSTRUCTIONS

This is a practical examination. Candidates should answer **ALL** questions

ADDITIONAL REQUIREMENTS

Mathematica version 13.0.1 software

MATLAB version 2016a software

2 GB Flash drive

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Answer ALL questions from this section (50 marks each).

A1. Use MATHEMATICA to answer question A1 and write the answers in the answer book provided. At the end of your MATHEMATICA session, save your worksheet under the name C: - - - .nb, where - - - are the last four digits of your student number.

(i) Given the matrices, $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

Compute

(a) BA [2]

(b) $2A^T + B$ [2]

(c) $tr(B^T A^T + 2C^T)$ [2]

(d) C^{-3} [2]

(ii) Let $\mathbf{u} = (1, 2, -3, 5, 0)$, $\mathbf{v} = (0, 4, -1, 1, 2)$, and $\mathbf{w} = (7, 1, -4, -2, 3)$. Find the components of

(a) $\frac{1}{2}(\mathbf{w} - 5\mathbf{v} + 2\mathbf{u}) + \mathbf{v}$ [3]

(b) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$ [3]

(iii) Let $\mathbf{u} = (3, 2, -1)$, $\mathbf{v} = (0, 2, -3)$, and $\mathbf{w} = (2, 6, 7)$. Find

(a) $\mathbf{u} \cdot \mathbf{v}$ [3]

(b) $\mathbf{u} \times \mathbf{v}$ [3]

(iv) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

[3, 3]

(v) On the same plane, plot the functions

$$\cos x + x^2, \quad 3 \cos^2 x + 2, \quad -\pi \leq x \leq \pi$$

[5]

(vi) Consider an IVP

$$y' + 2xy = 0, \quad y(0) = 1$$

Solve and plot the solution of the IVP. [5]

(vii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(a) $x^2(x - y)^2 = x^2 - y^2$ [3]

(b) $x \cos(2x + 3y) = y \sin x$ [3]

(viii) Evaluate

$$\lim_{x \rightarrow -\infty} \left(5 - \frac{2}{x^4} \right)$$

[2]

(ix) Evaluate

(a) $\int (\cos x \sin 2x + \sin x \cos 2x) dx$ [3]

(b) $\int_0^2 x^3 e^{x^4} dx$ [3]

A2. Use MATLAB to answer question A2 and write the answers in the answer book provided. At the end of your MATLAB session, save your worksheet under the name C: - - - - .m, where - - - - are the last four digits of your student number.

(i) Given the matrix

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 3 \\ 6 & 1 & 2 \end{bmatrix}$$

(a) Find the determinant of matrix A . [3]

(b) Find the inverse of matrix A . [3]

(c) Find the eigenvalues and eigenvectors of the matrix A . [4]

(ii) Suppose

$$y = x + \sin 2x, \quad \text{and} \quad y = \tan x - 4x, \quad \text{for} \quad -2\pi/3 \leq x \leq 2\pi/3$$

(a) Find the absolute maxima and minima of the function $y = x + \sin 2x$ in the interval $-2\pi/3 \leq x \leq 2\pi/3$. [4]

(b) Plot $y = x + \sin 2x$, and $y = \tan x - 4x$, for $-2\pi/3 \leq x \leq 2\pi/3$ on the same graph. Include the following

- labelled axes
- grid lines
- legend
- title
- clear distinction of the curves. [6]

(iii) Solve the odes in MATLAB

(a) $\frac{dy}{dt} = ty, \quad y(0) = 2$ [4]

(b) $\frac{d^2y}{dx^2} = \cos(2x) - y, \quad y(0) = 1, \quad y'(0) = 0$ [5]

(iv) Solve the ode and plot the solution

$$\frac{dy}{dt} + 4y(t) = e^{-t}$$

[7]

(v) Evaluate the integral

(a) $\int_0^\infty \frac{(\ln(x))^3}{x} dx$ [3]

(b) $\int_0^{\pi/2} \frac{\sin^2 x}{\sqrt{1-\cos x}} dx$ [3]

(vi) Consider the function

$$f(x, y) = \frac{1}{(\sqrt{x+y})(1+x+y)^2}$$

- (a) Plot the function $f(x, y)$ [3]
- (b) Integrate over the triangular region bounded by $0 \leq x \leq 1$ and $0 \leq y \leq 1 - x$ [5]

END OF QUESTION PAPER