



GWANDA STATE UNIVERSITY

CMS1201

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

CALCULUS II

EPOCH MINE CAMPUS

Ms B KWIRIRA

JUNE 2025: EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY TWO** questions from **Section B** (30 marks each).

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

A1. (a) Define what is **meant** by saying two vectors **A** and **B** are perpendicular. [2]

(b) Let $\mathbf{A} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j}$. Find a unit vector orthogonal to both **A** and **B**. [6]

A2. The epicycloid C is given by $\mathbf{r}(x) = (5 \cos x - \cos 5x)\mathbf{i} + (5 \sin x - \sin 5x)\mathbf{j}$, $0 \leq x \leq 2\pi$. Find the intervals in which C is smooth. [6]

A3. Determine the nature of stationery points for the function

$$f(a, b) = a^2 - 2ab + b^2 - 2a + 2b + 4.$$

[6]

A4. Let $f(x, y, z) = x^2 \ln(xyz) \sin(yz)$. Find the gradient of the function $f(x, y, z)$. [8]

A5. Evaluate $\int_1^4 \int_0^1 \int_0^x 2ze^{-x^2} dydx dz$. [6]

- A6.** Determine whether the series $\sum_{m=0}^{\infty} \frac{m^2 2^{m+1}}{3^m}$ converges or diverges. [6]

SECTION B: Answer ANY two questions [60].

- B7.** (a) Find a set of parametric equations of the line that passes through the points $P(-2, 1, 0)$ and $Q(1, 3, 5)$. [6]
- (b) Let $A = (1, 1, 1)$, $B = (0, 3, 1)$ and $C = (-1, 2, 4)$ be points on the plane P_1 . Find vectors \mathbf{AB} and \mathbf{AC} . Hence or otherwise, find the equation of the plane P_1 . [10]
- (c) Let P_2 be a plane with equation $2x + 3y + 4z = 5$. Find the line of intersection between P_1 in question B7(b) and the plane P_2 . [6]
- (d) A particle moves along the curve C given by $\mathbf{r}(t) = 2 \sin \frac{t}{2} \mathbf{i} + 2 \cos \frac{t}{2} \mathbf{j}$. Find in terms of t the velocity vector, acceleration vector and the speed of the particle. [3,3,2]

- B8.** (a) (i) Define what is **meant** by saying the function $f(x, y)$ converges to l as (x, y) tends to a point (x_0, y_0) . [2]
- (ii) Use the definition to show that $\lim_{(x,y) \rightarrow (2,3)} x^2 + 2y = 10$. [8]
- (b) Show that the function

$$f(x, y) = \begin{cases} \left(\frac{x^2 - y^2}{x^2 + y^2} \right)^2, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is not continuous at the point $(0, 0)$. [4]

- (c) Let $u = x + y$ and $v = xy$. Find $\frac{\partial x}{\partial u}$, $\frac{\partial x}{\partial v}$, $\frac{\partial y}{\partial u}$ and $\frac{\partial y}{\partial v}$. [8]
- (d) Find the equation of the tangent plane to the hyperboloid given by

$$z^2 = 2x^2 + 2y^2 + 12$$

at the point $(1, -1, 4)$. [8]

- B9.** (a) The spherical coordinates are given by $x = r \cos \theta \sin \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \phi$. Show that the Jacobian from (x, y, z) to (r, θ, ϕ) is given by $-r^2 \sin \phi$. [10]
- (b) Sketch the region of integration R of the double integral $\int \int_R (2x^2 y^{-2} + 2y) dy dx$, where R is bounded by $1 \leq x \leq 2$ and $1 \leq y \leq x$. Hence or otherwise evaluate $\int \int_R (2x^2 y^{-2} + 2y) dy dx$. [10]
- (c) Find $\int \int_R (x^2 + y^2) dx dy$, where R is the region bounded by $xy = 1$, $xy = 3$, $x^2 - y^2 = 1$ and $x^2 - y^2 = 4$. [10]