GWANDA STATE UNIVERSITY

Faculty of Computational Sciences

DEPARTMENT OF MATHEMATICS AND STATISTICS

Theory of Estimation

CMS 2202

Examination Paper

April 2025

This examination paper consists of 3 printed pages

Time Allowed: 3 hours

Total Marks: 100

Examiner's Name: Mr. E. Utete

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

Graph papers

Statistical Tables

Section A: Answer All Questions (40 Marks)

A1 (a) Define the term **estimator** and distinguish it from an **estimate**. Provide an example of each. [5]

(b) Explain the **method of moments** for parameter estimation. Derive the method of moments estimator for the mean of a normal distribution. [6]

(c) State the **Cramér-Rao inequality** and explain its significance in evaluating the efficiency of an estimator. [5]

(d) What is a **confidence interval**? How does it differ from a **credible interval** in Bayesian estimation? [3]

A2 (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with mean μ and variance σ^2. Show that the sample mean $\bar{X}$ is an unbiased estimator of μ. [6]

(b) Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables from a Poisson distribution with parameter λ. Find the maximum likelihood estimator (MLE) of λ. [7]

(c) A random sample of size $n = 25$ is drawn from a normal distribution with unknown mean μ and known variance $\sigma^2 = 16$. The sample mean is $\bar{x} = 50$. Construct a 95% confidence interval for μ. [4]

(d) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli distribution with parameter p. Find the method of moments estimator for p. [4]

Section B: Answer Any 3 Out of 4 Questions (60 Marks)

Each Question is Worth 20 Marks

B3 (a) Define the following properties of estimators:

i. Unbiasedness. [2]

ii. Efficiency. [2]

iii. Consistency. [2]

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with mean μ and variance σ^2. Show that the sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of σ^2. [7]

(c) Discuss the concept of **sufficiency** in estimation. State the **Factorization Theorem** and use it to show that $\bar{X}$ is a sufficient statistic for μ in a normal distribution with known variance. [7]

B4 (a) Explain the concept of a **confidence interval** and how it is constructed using the pivotal quantity method. [6]

(b) A random sample of size $n = 36$ is drawn from a normal distribution with unknown mean μ and unknown variance σ^2. The sample mean is $\bar{x} = 100$, and the sample standard deviation is $s = 15$. Construct a 99% confidence interval for μ. [7]

(c) Discuss the interpretation of confidence intervals. How does the sample size affect the width of a confidence interval? [7]

B5 (a) Explain the difference between **prior**, **likelihood**, and **posterior** distributions in Bayesian inference. [8]

(b) Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables from a Bernoulli distribution with parameter p. Assume a Beta(α, β) prior for p. Derive the posterior distribution of p and find the Bayesian point estimate (posterior mean). [7]

(c) Compare and contrast **Bayesian credible intervals** with **frequentist confidence intervals**. [5]

B6 (a) Define **robust estimation** and explain why it is important in statistical analysis. Provide an example of a robust estimator. [6]

(b) Discuss the concept of **nonparametric estimation**. How does it differ from parametric estimation? Provide an example of a nonparametric estimator. [8]

(c) A researcher wants to estimate the median income of a population. Suggest a suitable nonparametric method and explain how it can be applied. [6]

End of Examination Paper

Total: 100 marks.