GWANDA STATE UNIVERSITY

Faculty of Computational Sciences

DEPARTMENT OF MATHEMATICS AND STATISTICS

Hypothesis Testing

CMS 2102

Examination Paper

NOVEMBER 2024

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: Mr. E. Utete

<u>INSTRUCTIONS</u>

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

Graph papers

Statistical Tables

SECTION A : Answer ALL Questions 40 marks

A1 (a) Define the following terms

i. size of hypothesis test. [4]

ii. power of hypothesis test. [4]

(b) Let X be one observation from the density

$$f(x;\theta) = \theta(\theta+1)x^{\theta-1}(1-x) \ ; \quad 0 < x < 1$$

for testing

$$H_0 : \theta = 1 \quad against \quad H_1 : \theta = 2$$

We used the test that rejects H_0 if and only if $X > 0.12$.

i. Find the size of this test. [4]

ii. Find the power function of this test. [4]

A2 On a research on chemical process yield, it is known that $\sigma = 3$ and also that yield is normally distributed. If $n = 5$ observations on yield are 91.6%, 88.75%, 90.8%, 89.95%, and 91.3%. In testing the hypothesis $H_0 : \mu = 90\%$ against $H_1 : \mu \neq 90\%$ using $\alpha = 0.05$.

(a) What is the P-value for this test? [3]

(b) What sample size would be required to detect a true mean yield of 85% with probability 0.95? [5]

(c) What is the type II error probability if the true mean yield is 92%? [4]

A3 An engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is approximately normally distributed with $\sigma = 60$ psi. A random sample of 12 specimens has a mean tensile strength of $\bar{x} = 3250$ psi.

(a) Test the hypothesis that mean strength is 3500 psi using $\alpha = 0.01$. [5]

(b) What is the smallest level of significance at which you would be willing to reject the null hypothesis? [3]

(c) Explain how you could answer the question in part A3(a) with a two-sided confidence interval on mean tensile strength. [4]

SECTION B : Answer THREE QUESTIONS only : 60 marks

B4 Suppose $X_1; X_2; X_3; ...; X_n$ from a normal distribution $N(\mu; \sigma^2)$ where both μ and σ are unknown. We wish to test the hypotheses.

$$H_0 : \sigma^2 = \sigma_0^2 \quad vs. \quad H_1 : \sigma^2 \neq \sigma_0^2$$

at the level α. Find the likelihood ratio. [20]

B5 Suppose that we have a random sample of size n from the density function

$$f(x;\theta) = \frac{1}{2\theta^3} x^2 e^{\frac{-x}{\theta}}; x > 0$$

i. Derive the most powerful test of size α for testing

$$H_0 : \theta = 5 \quad against \quad H_1 : \theta = 3$$

[10]

ii. Find the Type II error for the test. [5]

iii. Write the p-value formula for a given value of $\bar{X} = \bar{x}$ [5]

B6 Consider a random sample $X_1; X_2; X_3; ...; X_n$ from a distribution with pdf

$$f(x;\beta) = \frac{1}{\beta} e^{-\frac{x-\theta}{\beta}}; \qquad \theta < x < 1; \quad \beta > 0 \qquad zero\ elsewhere.$$

where θ is known.

i. Derive a generalized likelihood ratio test of size α for testing

$$H_0 : \beta = \beta_0 \quad against \quad H_1 : \beta < \beta_0$$

[15]

ii. Find the power function for this test. [5]

B7 Let $X_1; X_2; X_3; ...; X_n$ be a random sample from a $Beta(\theta; 1)$ with pdf

$$f(x;\theta) = \theta x^{\theta-1}; 0 \leq x \leq 1; \theta > 0; zero\ elsewhere.$$

For testing

$$H_0 : \theta = 1 \quad against \quad H_1 : \theta = 2$$

i. Find the most powerful test of size α. [16]

ii. Explain why we don't say "*accept* H_0" when we cannot reject the null hypothesis. [4]