GWANDA STATE UNIVERSITY

ESG2212

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

NUMERICAL METHODS AND ORDINARY DIFFERENTIAL EQUATIONS

EPOCH MINE CAMPUS

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SEPTEMBER 2024: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY TWO** questions from **Section B** (30 marks each).

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

A1. (a.) Define the following terms as they are used in numerical methods;

(i) LU decomposition
(ii) Augumented matrix
(iii) Ordinary differential equation. [2,2,2]

b(i) Outline the steps when using the Bisection method. [6]

b(ii) The root of $e^x - 2 = 0$ is known to exist in $[0, 1]$. Use the Bisection method to find an approximate value of the root with $\epsilon = 0,01$. [9]

b(iii) List the strengths and limitations of using the Bisection method in solving non-linear equations. [4]

A2. (a.) List three elementary row operations. [3]

(b.) Solve the following system of equations using Gauss elimination

$$\begin{aligned} x + 2y + 2z &= 2 \\ 3x - 2y - z &= 5 \\ 2x - 5y + 3z &= -4 \\ x + 4y + 6z &= 0. \end{aligned}$$

[12]

SECTION B: Answer ANY TWO questions [60].

B3. (a.) solve using the Jacobi iteration method [15]

$$\begin{aligned} 10x_1 - x_2 + 2x_{3]} &= 6 \\ -x_1 - 11x_{2x3} + 3x_4 &= 25 \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11 \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$

(b.) Discuss the application of Error analysis in Geomatics and surveying. [7]

(c.) Use the trapezoidal rule with $n = 10$ to find the approximation to $\int_0^2 e^2 x$ correct to 5 decimal places. [8]

B4. (a.) Consider the following table;

1.3	1.4	1.45	1.5	1.55	1.6
4.77	5.677	6.182	6.723	7.3028	7.9248

Find $f^{'}(1,5)$ using;

(i) the Richard extrapolation method. [8]

(ii) the forward difference method. [8]

(iii) the central difference method. [8]

(iv) the backward difference method. [6]

B5. (a.) .Use the Euler's method to solve $y^{'} = \frac{(x-y)}{2}$ on $[0,3]$ with $y(0) = 1$ for $h = 1$. [10]

(b.) Use the modified Euler's method to solve $y^{'} = \frac{(x-y)}{2}$ on $[0,3]$ with $y(0) = 1$ for $h = 0.5$. [8]

(c.) Find $f"(0)$ for $f(x) = cosx$ where $h = 0,1$ and $h = 0,01$. Find the error associated with using these two step sizes and comment. [12]