# GWANDA STATE UNIVERSITY

## FACULTY OF COMPUTATIONAL SCIENCES

### DEPARTMENT OF GEOMATICS AND SURVEYING

**Probability Theory and Statistics**

## ESG 1111

**Semester TWO 2025/2026 : Main Examination**

This examination paper consists of 4 printed pages, inclusive of this cover page.

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|---|---|
| **Date:** | **June 2026** |
| **Total:** | **100 Marks** |
| **Time:** | **3 hours** |
| **Examiner's Name:** | **Mr. E. Utete** |

## INSTRUCTIONS

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY THREE** questions from **Section B** each question carries **20 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

## ADDITIONAL MATERIALS

- Calculator (non-programmable).
- Pens and Pencils.
- List Of Formulae and Statistical Tables.

**SECTION A: Answer ALL questions [40].**

**A1.** Define the following terms as used in statistics:

(a) Population **(3 Marks)**

(b) Sample **(3 Marks)**

(c) Statistic **(3 Marks)**

**A2.** A surveyor classifies land parcels as Residential (R), Commercial (C), or Agricultural (A). In a certain district, 50% are Residential, 30% are Commercial, and 20% are Agricultural.

(a) What is the probability that a randomly selected parcel is either Residential or Commercial? **(2 Marks)**

(b) If two parcels are selected independently, what is the probability that both are Agricultural? **(3 Marks)**

(c) What is the probability that one parcel is Residential and the other is Commercial? **(3 Marks)**

**A3.** The number of calibration errors ($X$) found in a GPS device per week has the following probability distribution:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X = x)$ | 0.4 | 0.4 | 0.2 |

(a) Calculate the expected number of errors per week, $E[X]$. **(4 Marks)**

(b) Calculate the variance of $X$, $Var(X)$. **(5 Marks)**

**A4.** A surveyor measures the same baseline 16 times using a new EDM device. The sample mean is $\bar{x} = 120.45$ m and the sample standard deviation is $s = 0.12$ m.

(a) Calculate the standard error of the mean. **(2 Marks)**

(b) Construct a 90% confidence interval for the true length of the baseline. **(4 Marks)**

**A5.** (a) State the difference between a Type I error and a Type II error in hypothesis testing. **(4 Marks)**

(b) State any FOUR properties of a normal distribution. **(4 Marks)**

**SECTION B: Answer ANY three questions [60].**

**B6.** A surveyor uses a total station to measure a distance 9 times.
The results (in meters) are:
45.23, 45.21, 45.25, 45.22, 45.24, 45.23, 45.22, 45.24, 45.23

(a) Calculate the sample mean of these measurements. **(4 Marks)**

(b) Calculate the sample variance and sample standard deviation. **(6 Marks)**

(c) Construct a 95% confidence interval for the true distance.
Assume the measurements are normally distributed. **(6 Marks)**

(d) Interpret what this confidence interval means. **(4 Marks)**

**B7.** A researcher compares the accuracy of two digitizing tablets.
The errors (in mm) for a sample of points are:

- Tablet A: 2.5, 3.0, 2.8, 3.2, 2.9 (n = 5)
- Tablet B: 2.1, 2.4, 2.3, 2.6, 2.2, 2.5 (n = 6)

(a) Calculate the sample mean and sample variance for each tablet. **(6 Marks)**

(b) Test if Tablet B has a smaller mean error than Tablet A.
Use $\alpha = 0.05$ and assume equal variances. **(10 Marks)**

(c) State your conclusion clearly. **(4 Marks)**

**B8.** A surveyor is studying the relationship between the distance from a reference point ($X$, in km) and the elevation ($Y$, in meters). Data from 6 points are:

| $X$ (km) | $Y$ (m) |
|---|---|
| 1 | 120 |
| 2 | 135 |
| 3 | 148 |
| 4 | 162 |
| 5 | 175 |
| 6 | 190 |

Summary statistics:
$\sum x = 21$, $\sum y = 930$, $\sum x^2 = 91$, $\sum y^2 = 147\ 498$, $\sum xy = 3\ 497$

(a) Calculate the Pearson correlation coefficient $r$. **(5 Marks)**

(b) Interpret the value of $r$. **(3 Marks)**

(c) Find the equation of the regression line $\hat{y} = b_0 + b_1 x$. **(7 Marks)**

(d) Predict the elevation at a distance of 5.5 km from the reference point. **(5 Marks)**

**B9.** The heights of trigonometric beacons in a certain region are approximately normally distributed with a mean of 850 meters and a standard deviation of 40 meters.

(a) What proportion of beacons have heights between 800 m and 900 m? **(5 Marks)**

(b) What is the probability that a randomly selected beacon has a height less than 780 m? **(4 Marks)**

(c) If a beacon is selected at random, find the height such that only 5% of beacons are taller than this height. **(5 Marks)**

(d) A surveyor randomly selects 9 beacons. What is the probability that the sample mean height exceeds 870 meters? **(4 Marks)**

(e) What assumption did you make in part (d) about the distribution of beacon heights? **(2 Marks)**

*End of questions : Total 100 marks*