GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

Theory of Estimation

CMS 2202

Semester Two 2025/2026 : Main Examination

This examination paper consists of 4 printed pages, inclusive of this cover page.

Date:	**June 2026**
Total:	**100 Marks**
Time:	**3 hours**
Examiner's Name:	**Mr. E. Utete.**

INSTRUCTIONS

Candidates should attempt **ALL** questions from **Section A** which carries **40 marks**. Answer **ANY THREE** questions from **Section B** each question carries **20 marks**. **Section B** carries **60 marks**.

- All answers and all rough work must be written in the answer booklet(s) provided.
- A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be graded.
- Black and blue pen only must be used for written answers and pencil for all drawings and sketches.

ADDITIONAL MATERIALS

- Calculator (non-programmable).
- Pens and Pencils.
- List Of Formulae and Statistical Tables.

SECTION A: Answer ALL questions [40].

A1. (a) Define the term **sufficient statistic** and explain why sufficient statistics are important in estimation theory. **(5 Marks)**

(b) State the Neyman-Fisher Factorization Theorem and Use it to find a sufficient statistic for the parameter θ based on a random sample $X_1, X_2, \ldots, X_n$ from a Poisson distribution with parameter θ. **(6 Marks)**

(c) Distinguish between a **sufficient statistic** and a **minimal sufficient statistic**. **(4 Marks)**

(d) Explain the concept of a complete sufficient statistic and discuss why the property of completeness is desirable for a sufficient statistic in statistical inference. **(5 Marks)**

A2. (a) Define the **bias** of an estimator $\hat{\theta}$ for a parameter θ and eplain when is an estimator said to be unbiased? **(4 Marks)**

(b) Define the **mean squared error** (MSE) of an estimator $\hat{\theta}$ for a parameter θ and show that:

$$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2.$$

(6 Marks)

(c) Two estimators, $\hat{\theta}_1$ and $\hat{\theta}_2$, are proposed for a parameter θ. Their properties are:

- $\hat{\theta}_1$ is unbiased with variance 8.
- $\hat{\theta}_2$ has bias 1 and variance 4.

(i) Compute the MSE for each estimator. **(3 Marks)**

(ii) Based on MSE, which estimator is preferable? Justify your answer. **(2 Marks)**

(d) How is the Cramér-Rao Lower Bound used to determine if an unbiased estimator is efficient? **(5 Marks)**

SECTION B: Answer ANY three questions [60].

B3. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with probability density function:

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 < x < 1,\ \theta > 0.$$

(i) Find the maximum likelihood estimator (MLE) of θ. **(7 Marks)**

(ii) Find the method of moments estimator of θ. **(6 Marks)**

(b) A random sample of size $n = 30$ is drawn from a normal distribution with unknown mean μ and unknown variance σ^2. The sample mean is $\bar{x} = 45$ and the sample standard deviation is $s = 8$.
Construct a 95% confidence interval for μ. **(5 Marks)**

(c) Interpret the confidence interval constructed in part (b). **(2 Marks)**

B4. (a) Explain the difference between the **prior distribution**, the **likelihood function**, and the **posterior distribution** in Bayesian inference. **(5 Marks)**

(b) Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables from a Bernoulli distribution with parameter p. Assume a Beta(α, β) prior for p.

(i) Derive the posterior distribution of p. **(8 Marks)**

(ii) Find the Bayesian point estimate of under quadratic loss (the posterior mean). **(3 Marks)**

(c) Compare and contrast Bayesian **credible intervals** with frequentist **confidence intervals**. **(4 Marks)**

B5. (a) Define the property of **consistency** for an estimator and explain why is consistency considered an asymptotic property? **(4 Marks)**

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with mean μ and variance $\sigma^2 < \infty$. Prove that the sample mean $\bar{X}$ is a consistent estimator of μ. **(6 Marks)**

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution on the interval $[0, \theta]$. Consider two estimators for θ:

- $\hat{\theta}_1 = 2\bar{X}$
- $\hat{\theta}_2 = X_{(n)}$, the maximum of the sample.

(i) Show that both estimators are unbiased estimators of θ. **(4 Marks)**

(ii) Which estimator is more efficient? Justify your answer. **(6 Marks)**

B6. (a) Define **robust estimation** and explain why robust estimators are important when analyzing real-world data. **(4 Marks)**

(b) A dataset contains the following values representing household incomes (in thousands of dollars):

$$22, 25, 28, 30, 32, 35, 38, 42, 45, 48, 52, 55, 58, 62, 120.$$

(i) Calculate the sample mean and the sample median. **(4 Marks)**

(ii) Which estimator provides a better measure of central tendency for this dataset? Explain your reasoning. **(4 Marks)**

(c) Explain the concept of **nonparametric estimation** and how does it differ from parametric estimation? **(4 Marks)**

(d) Describe the **empirical distribution function** (EDF) and explain how it can be used to estimate the probability $P(X \leq x)$ for any value x. **(4 Marks)**

End of Examination : Total 100 marks.