GWANDA STATE UNIVERSITY

Faculty of Computational Sciences

DEPARTMENT OF MATHEMATICS AND STATISTICS

Statistical Inference I

SMS 1104

Examination Paper

April 2024

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: Mr. E. Utete

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

Graph papers

Statistical Tables

SECTION A : Answer ALL Questions 40 marks

A1 Show that $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$, is a biased estimator of σ^2 [8]

A2 A synthetic fiber used in manufacturing carpet has tensile strength that is normally distributed with mean 75.5 psi and standard deviation 3.5 psi. Find the probability that a random sample of $n = 6$ fiber specimens will have sample mean tensile strength that exceeds 75.75 psi. [7]

A3 Explain the meaning of

(a) power of test. [4]

(b) critical region for a test. [4]

(c) p-value. [4]

(d) significance level. [4]

A4 Let

$$f(x) = \theta x^{1-\theta}, \qquad 0 < \theta < \infty, \qquad 0 < x < 1$$

Show that

$$\hat{\Theta} = \frac{-n}{\ln \prod_{i=1}^{n} X_i}$$

is the maximum likelihood estimator for θ [9]

SECTION B : Answer THREE QUESTIONS only : 60 marks

B5 (a) Explain the meaning of tolerance interval. [4]

(b) The diameter of holes for cable harness is known to have a normal distribution with $\sigma = 0.01$ inch. A random sample of size 10 yields an average diameter of 1.5045 inch. Find a 99% two-sided confidence interval on the mean hole diameter. [8]

(c) A study is to be conducted of the percentage of homeowners who own at least two television sets. How large a sample is required if we wish to be 99% confident that the error in estimating this quantity is less than 0.017? [8]

B6 Medical researchers have developed a new artificial heart constructed primarily of titanium and plastic. The heart will last and operate almost indefinitely once it is implanted in the patient's body, but the battery pack needs to be recharged about every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of these batteries is 4.05 hours. Assume that battery life is normally distributed with standard deviation $\sigma = 0.2$ *hour*.

(a) Is there evidence to support the claim that mean battery life exceeds 4 hours? Use $\alpha = 0.05$. [10]

(b) Compute the power of the test if the true mean battery life is 4.5 hours. [5]

(c) What sample size would be required to detect a true mean battery life of 4.5 hours if we wanted the power of the test to be at least 0.9? [5]

B7 A manufacturer of interocular lenses is qualifying a new diamond grinding machine and will qualify the machine if the percentage of polished lenses that contain surface defects does not exceed 2%. A random sample of 250 lenses contains six defective lenses.

(a) Formulate and test an appropriate set of hypotheses to determine if the machine can be qualified. Use $\alpha = 0.05$. [14]

(b) Find the P-value for the test. [6]

B8 A study was performed to determine whether men and women differ in their repeatability in assembling components on printed circuit boards. Random samples of 25 men and 21 women were selected, and each subject assembled the units. The two sample standard deviations of assembly time were $s_{men} = 0.98 minutes$ and $s_{women} = 1.02 minutes$.

(a) Is there evidence to support the claim that men and women differ in repeatability for this assembly task? Use $\alpha = 0.02$ [15]

(b) State any necessary assumptions about the underlying distribution of the data. [5]