GWANDA STATE UNIVERSITY

Faculty of Computational Sciences

DEPARTMENT OF MATHEMATICS AND STATISTICS

PROBABILITY THEORY 1

SMS 1112

Examination Paper

NOVEMBER 2023

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: 100

Examiner's Name: Mr. E. Utete

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

Graph papers

Statistical Tables

SECTION A : Answer ALL Questions 40 marks

A1 If the last digit of a mineral ore weight measurement is equally likely to be any of the digits 0 through 9,

(a) What is the probability that the last digit is 0? [2]

(b) What is the probability that the last digit is 3 or 7? [2]

(c) What is the probability that the last digit is greater than or equal to 5? [2]

A2 Suppose A and B are mutually exclusive events. Construct a Venn diagram that contains the three events A, B, and C such that $P(A \mid C) = 1$ and $P(B \mid C) = 0$ [5]

A3 (a) Explain the meaning of "lack of memory property of an exponential random variable" [4]

(b) Given the following Cumulative Distribution Function.

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.25x + 0.5, & -2 \leq x < 1 \\ 0.5x + 0.25, & 1 \leq x < 1.5 \\ 1, & 1.5 \leq x \end{cases}$$

i. Find the probability distribution function. [6]

ii. Make a sketch of the probability distribution function. [4]

iii. Find $P(-0.5 \leq x < 1.25)$ [4]

iv. $E(X)$ [4]

v. $E(X^2)$ [4]

vi. $Var(X)$ [3]

SECTION B : Answer THREE QUESTIONS only : 60 marks

B4 (a) Given the following probability mass functions,

$$f(x) = \frac{2x+1}{25}, x = 0, 1, 2, 3, 4$$

Find

i. $P(2 \leq X < 4)$ [2]

ii. $E(X)$ [3]

iii. $E(X^2)$ [4]

iv. $Var(X)$ [2]

(b) The number of failures of a mine water pump follows a Poisson random variable with a mean of 0.02 failure per hour.

i. What is the probability that the water pump does not fail in an 8-hour shift? [4]

ii. What is the probability of at least one failure in a 24-hour day? [5]

B5 The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 400 hours.

(a) What is the probability that an assembly on test fails in less than 100 hours? [4]

(b) What is the probability that an assembly on test fails between 50 and 100 hours? [4]

(c) What is the probability that an assembly operates for more than 500 hours before failure? [6]

(d) If an assembly has been on test for 400 hours without a failure, what is the probability of a failure in the next 100 hours? [6]

B6 (a) State four properties of a Normal Distribution. [4]

(b) The line width of for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

i. What is the probability that a line width is greater than 0.62 micrometer? [5]

ii. What is the probability that a line width is between 0.47 and 0.63 micrometer? [5]

iii. Find the minimum value of line width of semiconductor that contain the least 90% of the sample ? [6]

B7 (a) Determine the value of c such that the function $f(x, y) = cx^2y$ for $0 < x < 3$ and $0 < y < 2$ satisfies the properties of a joint probability density function. [5]

(b) The percentage of people given an antirheumatoid medication who suffer severe, moderate, or minor side effects are 10, 20, and 70%, respectively. Assume that people react independently and that 20 people are given the medication. Determine the following:

i. The probability that 2, 4, and 14 people will suffer severe, moderate, or minor side effects, respectively [3]

ii. The probability that no one will suffer severe side effects [3]

iii. What is the conditional probability distribution of the number of people who suffer severe side effects given that 19 suffer minor side effects? [5]

iv. What is the conditional mean of the number of people who suffer severe side effects given that 19 suffer minor side effects? [4]