GWANDA STATE UNIVERSITY

FACULTY OF COMPUTATIONAL SCIENCES

LINEAR MATHEMATICS 1

SMS 1111

This examination paper consists of 4 pages

Date:	**SEPTEMBER 2023**
Total Marks:	**100**
Time:	**2 hours**
Examiner's Name:	**Ms B. Kwirira**

INSTRUCTIONS

This paper consists of nine questions. Answer all questions in section A and answer any TWO questions in section B.

Use of calculator is permissible

SECTION A: Answer ALL questions [40].

A1. (*a*) Show that if $z_1, z_2 \in \mathbb{C}$, then $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$. [5]

(*b*) Given that $z = x + iy$, express the equation $2x + y = 5$ in terms of z and $\bar{z}$. [5]

A2. (a) Find the inverse of $\mathbf{A} = \begin{pmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{pmatrix}$. [7]

A3. (*a*) Let $f(x) = x^5 - 11x^3 - 26x^2 + 48x + 144$. Given that $x = -2 + 2i$ and $x = -2$ are roots of $f(x)$, find all the other roots of $f(x)$ and write $f(x)$ as a product of irreducible r eal quadratic and linear functions. [8]

A4. Given $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \mathbf{B} = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{C} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, find the magnitudes of (i) $\mathbf{A} + \mathbf{B} + \mathbf{C}$ and (ii) $2\mathbf{A} - 2\mathbf{B} - 5\mathbf{C}$. [5]

A5. (a) Explain when is the equation exact? [4]

(b) Solve the equation

$$(1 - \mathbf{sin\ x\ tan\ y})dy + (cosxsec^2y)dy = 0)$$

[6]

SECTION B (60 marks)

Candidates may attempt THREE questions being careful to number them B? to B?.

B6. (a) Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$, where $z \in \mathbb{C}$. [8]

(b) By writing $(-1 + i)$ in polar form, show that $(-1 + i)^7 = -8(1 + i)$. [8]

(i) Let A be an invertible square matrix. Prove that $(A^t)^{-1} = (A^{-1})^t$. [4]

(ii) Prove that, for any matrices A, B and C, the following holds for addition $A + (B + C) = (A + B) + C$. [5]

(c) Given that

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \quad \textit{and} \quad \mathbf{v_3} = \begin{bmatrix} -1 \\ 8 \\ -4 \end{bmatrix},$$

. Calculate $4\mathbf{v_1} - 2\mathbf{v_2} + \mathbf{v_3}$ [5]

B7. (a) For which values of α and β does the following system have a unique solution?

$$\begin{aligned} 2x + y + \alpha z &= \beta \\ 2x - \alpha y + 5z &= \beta \\ x - y + \alpha z &= 1. \end{aligned}$$

[15]

(b) Solve the following system of linear equations using Gauss elimination

$$\begin{aligned} x + 2y + 2z &= 2 \\ 3x - 2y - z &= 5 \\ 2x - 5y + 3z &= -4 \\ x + 4y + 6z &= 0. \end{aligned}$$

[15]

B8. (a) Show that $|z_1 + z_2| \leq |z_1| + |z_2|$ for any $z_1, z_2 \in \mathbb{C}$. [7]

(b) Solve $xy' + 4y = x^6$ [5]

(c) Solve $y' - \frac{y}{x} = \frac{5}{2}x^2y^3$ [8]

(d) Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Express the vector $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 10 \end{bmatrix}$ as a linear combination $\mathbf{v_1}, \mathbf{v_2}$ and $\mathbf{v_3}$. [10]

B9. (a) Given that $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$.

(i) Calculate $A^t A$ and what type of matrix is $A^t A$?

(ii) Does A^t and A commute? [4,4]

(b) Solve $y'' + 3y' - 10y = 0$ [6]

(c) Find the general solution for $y'' - 6y' + 9y = 0$ [5]

(d) A square matrix P is idempotent if $P^2 = P$.

(i) Suppose P is a square matrix of order m and let I be the $m \times m$ identity matrix. Prove that them matrix $I - P$ is an idempotent matrix. [5]

(ii) Assume that P is an $m \times m$ nonzero idempotent matrix. Find all integers k such that the matrix $I - kP$ is idempotent. [6]

END OF QUESTION PAPER