GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

MECHANICS (ESG 1209)

Final Examination Paper

November 2023

EPOCH MINE CAMPUS

Time Allowed: **3 hours**
Total Marks: **100**
Examiner's Name: **Mr. P. Sigwegwe**

INSTRUCTIONS

1. Answer **ALL** question in SECTION A.
2. Answer any **THREE** questions from SECTION B.
3. Use of calculators is permissible.

MARK ALLOCATION

Section A	**40**
Question A1	20
Question A2	20
Section B	**60**
Question B3	20
Question B4	20
Question B5	20
Question B6	20
Total Attainable	**100**

SECTION A ANSWER ALL QUESTIONS IN THIS SECTION (40 Marks)

Question A1

a) A block of mass m is attached to a spring with spring constant k and is free to slide along a horizontal frictionless surface. At $t = 0$, the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest, $v_{x,0} = 0$.

i. What is the period of oscillation of the block?

ii. What is the velocity of the block when it first comes back to the equilibrium position?

iii) Show that the total energy of the system mid-way the equilibrium position and maximum displacement is, $\frac{1}{2}kx_0^2$ **(10)**

Question A2

a) State Newton's Second Law of motion and briefly describe how it can be used to define a Newton **(2)**

b) The definition of velocity states that $\boldsymbol{v} = \frac{\boldsymbol{dx}}{\boldsymbol{dt}}$

Show that the definition of velocity can be used to derive the equations of motion:

$$x = v_0 t + \frac{1}{2}at^2 + C',$$

Where C is a constant of integration. **(4)**

c) Consider an object of mass m that is in free fall but experiencing air resistance. The magnitude of the drag force is given by,

$$F_{\text{drag}} = \frac{1}{2}C_D A\rho v^2$$

Where **ρ** is the density of air, ***A*** is the cross-sectional area of the object in a plane perpendicular to the motion, ***v*** velocity and $\mathbf{C}_D$ is the drag coefficient. Assume that the object is released from rest and very quickly attains speeds in which the above equation applies.

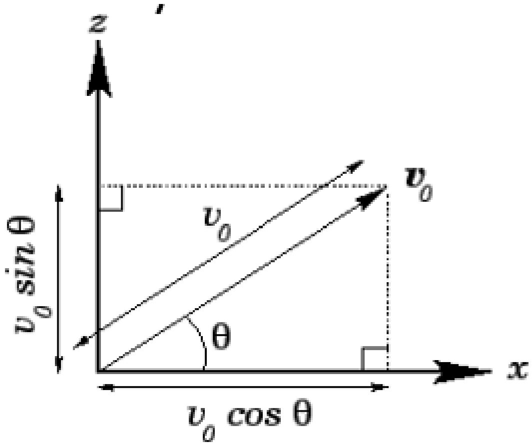
With the aid of simple and clear diagrams determine

(i) the terminal velocity, and **(2)**

(ii) the velocity of the object as a function of time. **(2)**

Question A3

a) The diagram below shows the vector decomposition of the initial conditions of projectile motion



(Figure A3.0)

Using the diagram above and any other relevant classical mechanics assumptions derive the following expressions

i. $$v_0 = (v_{x,0}^2 + v_{y,0}^2)^{1/2}$$ **(2)**

ii. $$R = \frac{v_0^2}{g} \sin 2\theta_0.$$ **(2)**

b) State Newton's Law of Gravitation **(1)**

c) Using the law stated above derive an equation for gravitational potential **(2)**

d) One of Kepler's laws of planetary motion relates the period and radius, state the Law and derive it **(3)**

Question A4

a) A 68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15 degree above the horizontal.

  i. If the coefficient of static friction is 0.50, what minimum force magnitude is required from the rope to start the crate moving? **(2)**

  ii. If μk=0.35, what is the magnitude of the initial acceleration of the crate? **(2)**

b) State the principle of conservation of energy **(1)**

c) State the principle of linear conservation of momentum **(1)**

d) One of the most important examples of periodic motion is simple harmonic motion (SHM), in which some physical quantity varies sinusoidal. Suppose a function of time has the form of a sine wave function,

$$y(t) = A\sin(wt)$$

For the harmonic oscillator stated above derive an expression for

  i. Velocity **(1)**

ii. Acceleration **(1)**

iii. Kinetic energy **(2)**

SECTION B (60 marks)

Answer ANY THREE questions from this section.

Question B5

Suppose $x_1(t)$ and $x_2(t)$ are both solutions of the simple harmonic oscillator equation.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x\,.$$

By *ansatz* (educated guess) the linear combination $x(t) = x_1(t) + x_2(t)$ is also a solution of the SHO equation,

$x_1(t) = D\sin(\omega_0 t)$,

$x_2(t) = C\cos(\omega_0 t)$

b) Find the linear combination $x(t) = x_1(t) + x_2(t)$ **(1)**

c) Determine the velocity of the linear combination

i. $v_{(x)}$ **(2)**

ii. $a_{(x)}$ **(2)**

d) Show that the linear combination of the two solutions in also a solution to the simple harmonic oscillator equation. **(5)**

e) A block of mass m is attached to a spring with spring constant k and is free to slide along a horizontal frictionless surface. At $t = 0$, the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest, $v_{x,0} = 0$.

iii. What is the period of oscillation of the block?

iv. What is the velocity of the block when it first comes back to the equilibrium position?

v. Show that the total energy of the system mid-way the equilibrium position and maximum displacement is,

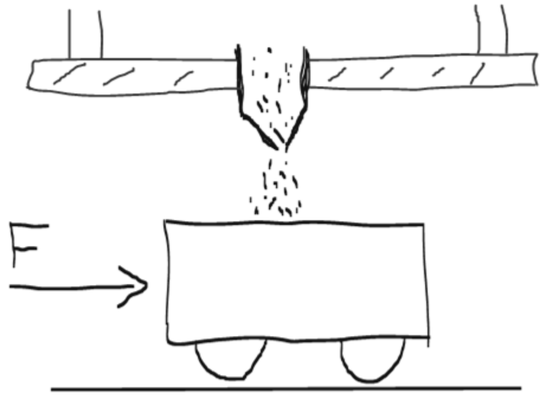
$$= \frac{1}{2}k x_0^2\left(\cos^2\left(\sqrt{\frac{k}{m}}t\right) + \sin^2\left(\sqrt{\frac{k}{m}}t\right)\right)$$

(10)

Question B6

a) Define impulse? **(1)**

b) An empty coal car of mass m_0 starts from rest under an applied force of magnitude F. At the same time coal begins to run into the car at a steady rate *b* from a coal hopper at rest along the track (Figure B6.0). Find the speed when a mass m_c of coal has been transferred **(6)**



c) A high pressure hose is being used to a storage tank the hose delivers a horizontal stream of water which hits the wall at a speed of 20 m/s.
Find the average force exerted on wall, assuming that the water does not bounce back off the wall, if

i. 8 kg of water is delivered per second, **(3)**

ii. the cross-sectional area of the hose pipe is 0,5 cm2. (Take the density of water as 1000 kg/m3) **(4)**

d) Given that the body of mass m starts from rest and reaches a speed v after moving through a distance s under the action of a constant force, F, show that kinetic energy (Ek) is given by:

$$E_k = \frac{1}{2}mv^2$$

(6)

<u>Question B7</u>

7.1 Figure B7.0 shows a train of four blocks being pulled across a frictionless floor by force What total mass is accelerated to the right by:

(a) force ,

(b) cord 3, and

(c) cord 1?

(d) Rank the blocks according to their accelerations, greatest first.

(e) Rank the cords according to their tension, greatest first**.** **[5]**

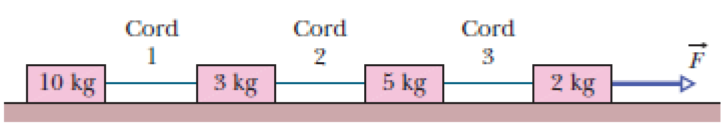


Figure B7.0

7.2 In Fig. B7.1, let the mass of the block be 8.5 kg and the angle be Ø=30 degrees.
Find

(a) the tension in the cord and

(b) the normal force acting on the block.

(c) If the cord is cut, find the magnitude of the resulting acceleration of the block. **[6]**

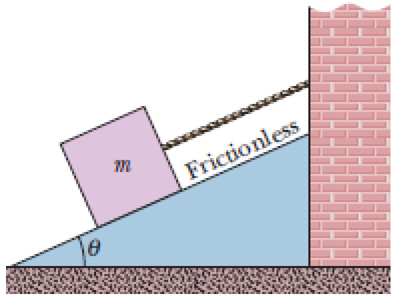


Figure B7.1

7.3 Figure 7.2 shows a *Conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. (The cord sweeps out a cone as the bob rotates.) The bob has a mass of 0.040 kg, the string has length L =0.90 m and negligible mass, and the bob follows a circular path of circumference 0.94 m.

What are:

a) the tension in the string and

b) the period of the motion**?** **[6]**

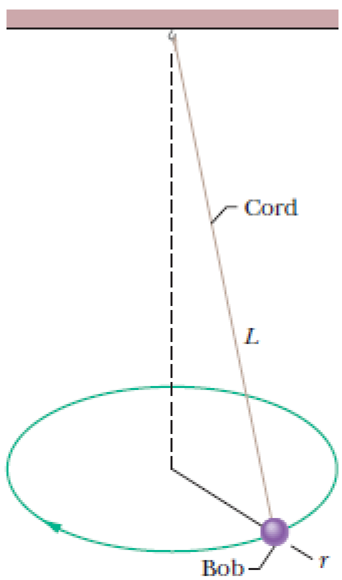


Figure B7.2

7.4 State any three ways in which heavy duty industrial machines can produce more work output in relation to the energy input **[3]**

<u>Question B8</u>

a) State Archimedes' principle **(2)**

b) a block of density 800 kg/m3 floats face down in a fluid of density 1200 kg/m3, The block has height of H =6.0 cm

   i. By what depth h is the block submerged? **(4)**

c) State and explain four assumptions that we make about our ideal fluid; they all are concerned with flow. **(8)**

d) While doing field work a Surveyor runs away from wasps in an open field on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the Surveyors's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

a) Find the following

i. The velocity vector components in the *x y and* z directions **(2)**
ii. The acceleration vector components in the *x y and z* directions **(2)**
iii. Magnitude of velocity and its direction **(1)**
iv. Magnitude of acceleration and its direction **(1)**

<u>End of Question Paper.</u>