GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF GEOMATICS AND SURVEYING

ENGINEERING MATHEMATICS II

EMR 1201, EMI 1201

This examination paper consists of 4 pages

Date:	**May 2023**
Total Marks:	**100**
Time:	**3 hours**
Examiner's Name:	**Mr. M. Mpofu**

INSTRUCTIONS

This paper consists of Section A (40 marks) and Section B (60 marks). Answer **ALL** questions in **Section A** and answer **ANY THREE** questions in **Section B**.

Use of calculator is permissible

ADDITIONAL MATERIALS

- Calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Find the steady state motion of the mass-spring system modeled by the ODE

$$y'' + 2.5y' + 10y = -13.6 \sin 4t$$

[7]

A2. Solve

(a) $x\frac{dy}{dx} - 4y = x^6 e^x$ [4]

(b) $(1 + \ln x + \frac{y}{x}dx = (1 - \ln x)dy$ [5]

A3. A heart pacemeter consists of a switch, a battery of constant voltage $E_\circ$, a capacitor with constant capacitance C, and the heart as a resistor with constant resistance R. When the switch is closed, the capacitor charges; when the switch is open, the capacitor discharges, sending an electrical stimulus to the heart. During the time the heart is being stimulated, the voltage E across the heart satisfies the linear differential equation

$$\frac{dE}{dt} = -\frac{1}{RC}E$$

Solve the differential equation subject to $E(4) = E_\circ$ [5]

A4. Solve the boundary value problem

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}$$

given that $u(x, 0) = 4e^{-3x} + 8e^{-5x}$. [6]

A5. (a) Find the Fourier series of $f(x) = sinx$, $\frac{-\pi}{2} \leq x < \frac{\pi}{2}$; where $f(x + \pi) = f(x)$. [5]

(b) (i) Find the Fourier half-range cosine series for the function defined by

$$f(x) = x, \qquad 0 \leq x \leq 1.$$

[5]

(ii) Sketch the function represented by the series over the interval $-3\pi \leq x \leq 3\pi$. [3]

SECTION B (60 marks)

Answer ANY THREE questions from this section.

A6. (a) Define a linear n^{th}-order ODE. [2]

(b) Find the general solution of $5y\cos^3 4xdy + \sin 3xdx = 0$ [4]

(c) Given an RLC-circuit modeled by

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = E_\circ \omega \cos \omega t,$$

with $R = 11\ \Omega$, $L = 0.1\ H$ (Henry), $C = 10^{-2}\ F$ (Farad), which is connected to a source of EMF $E(t) = 110\sin(60, 2\pi t)$ (hence $60\ \ Hz = 60\ \ cycles/sec$, the usual in the USA and Canada; in Europe it would be $220\ \ V$ and $50\ \ Hz$). Find the current $I(t)$ in the RLC-circuit, assuming that current and capacitor charge are 0 when $t = 0$ [14]

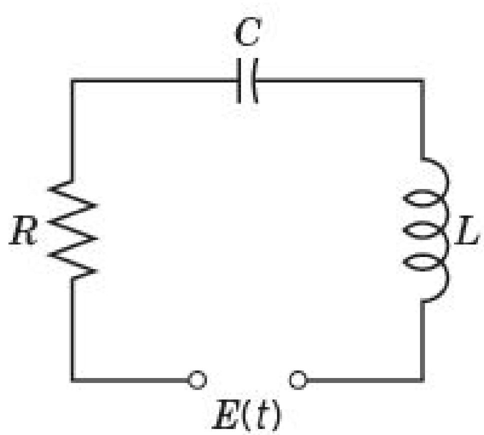

Figure 1: RLC-circuit

A7. (a) State **three** types of Fourier transforms. [3]

(b) Find the Fourier transform of

$$f(x) = H(x)e^{-kx}\cos(vx)$$

[8]

(c) Find the fourier series of

$$f(x) = \begin{cases} x & -\pi \le x < 0, \\ -x & 0 \le x < \pi \end{cases}$$

where $f(x) = f(x + 2\pi)$ [9]

A8. (a) Show that if initially $P = P_\circ$ and

$$\frac{dP}{dt} = kP - lP^2$$

where l and k are constants, then

$$P = \frac{kP_\circ}{lP_\circ + \alpha e^{-kt}}$$

where $\alpha = k - lP_\circ$ [5]

(b) Consider a mass-spring system with external force $r(t) = F_\circ \cos \omega t$, for $F_\circ > 0$ and $\omega > 0$ called the forcing function or the driving force. Suppose the forced motion of the mass-spring system is modeled by

$$my'' + cy' + ky = F_\circ \cos \omega t$$

Show that the particular solution ($y_p(t)$) is given by

$$y_p(t) = F_\circ \frac{m\left(\omega_\circ^2 - \omega^2\right)\cos \omega t}{m^2\left(\omega_\circ^2 - \omega^2\right) + \omega^2 c^2} + F_\circ \frac{\omega c \sin \omega t}{m^2\left(\omega_\circ^2 - \omega^2\right) + \omega^2 c^2}$$

where $\sqrt{\frac{k}{m}} = \omega_\circ$ or $k = m\omega_\circ^2$ [15]

A9. (a) Find the Laplace transforms of

(i) $f(t) = 3 + 2e^{-t}$ [3]

(ii) $f(t) = t^n$ [5]

(b) Solve the IVP using the Laplace transforms

$$\frac{d^2y}{dt^2} + y = 1, \qquad y(0) = 2, \qquad y'(0) = 0$$

[12]

A10. (a) Define a Boundary Value Problem (BVP). [3]

(b) Solve

$$\frac{d^2i}{dt^2} + 10i = V(t)$$

where $V(t) = \begin{cases} 1 & -\pi < t \leq 0 \\ -1 & 0 < t \leq \pi \end{cases}$ and $V(t + 2\pi) = V(t)$ [17]

END OF QUESTION PAPER