GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS III

EMN/EMG 2101

Examination Paper

NOVEMBER 2023

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: **100**

Examiner's Name: **Mr. R. G. Moyo**

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A(40 marks)

A1. Define the following terms as they are used in Numerical methods

(a) Truncation error [2]

(b) Optimization [2]

(c) Upper triangular matrix [2]

(d) Linear programming problem [2]

A2. (a) Outline the steps followed when solving a non-linear equation using the bisection method. [4]

(b) Use the Bisection Method to find the root of $f(x) = cosx - \sqrt{x}$ lying between $[0, 1]$. Use a tolerance of $\epsilon = 0.001$ and give your answer correct to 5 decimal places. [6]

A3. Evaluate $\int_0^1 e^{2x}dx$ using Simpson's rule. Use N=10 giving your answers correct to 5 decimal places. [6]

A4. Let $f(x)$ be given by the table below

x	1.2	1.4	1.45	1.5	1.55	1.6	1.7
$f(x)$	4.77009	5.67728	6.1152	6.18152	7.30278	7.92485	8.20577

Find an approximation to

(a) $f'(1.4)$ using the forward difference method [3]

(b) $f'(1.4)$ using the central difference method [3]

(c) $f'(1.5)$ using the backward difference method using $h = 0.1$ [3]

(d) $f''(1.6)$ [3]

(e) $f'(1.5)$ using the central difference method and compare it with the true value $f'(1.5) = 11.2042$ [4]

SECTION B (60 marks)

A5. (a) Write down a Matlab code to solve an initial value problem $y' = \frac{x-y}{2}$ on $[0; 100]$ using h=0.1 [8]

(b) Estimate the natural logarithm of 2 using linear interpolation. First, perform the computation by interpolating between $\ln 1 = 0$ and $\ln 6 = 1.791759$. Then repeat the procedure, but use a smaller interval, $\ln 1$ to $\ln 4 = 1.386294$. Compare the two estimates with the exact value of $\ln 2$ and give a comment. Note that the true value of $\ln 2$ is 0.6931472. [7]

(c) Discuss the application of interpolation in a mine setup. [5]

A6. (a) What is a matrix? [2]

(b) State any 3 methods that can be used to solve systems of linear equations. [3]

(c) Consider the following system of equations

$x_1 + x_2 + 3x_4 = 4$
$2x_1 + x_2 - x_3 + x_4 = 1$
$3x_1 - x_2 - x_3 + 2x_4 = -3$
$-x_1 + 2x_2 + 3x_3 - x_4 = 4$

(i) Write down the system in the form $\mathbf{A}\overline{x} = \mathbf{b}$ [3]

(ii) Find the inverse of the matrix $\mathbf{A}$ using elementary row operations [8]

(iii) Hence use the inverse to solve the above systems of equations. [4]

A7. A company manufactures two products of A and B. These products are processed in the same machine. It takes 10 minutes to process one unit of product A and 2 minutes for each unit of product B and the machine operates for a maximum of 35 hours in a week. Product A requires 1 kg and B requires 0.5 kg of raw material per unit, the supply of which is 600 kg per week. Market constraint on product B is known to be minimum of 800 units every week. Product A cost \$5 per unit and sold at \$10. Product B costs \$6 per unit and can be sold in the market at a unit price of \$8.

(a) Formulate a Linear programming problem for the above scenario. [4]

(b) Find the dual of the above Linear programming model [3]

(c) Solve the primal problem using the simplex algorithm. [13]

A8. (a) Solve the initial value problem $y' = \frac{x-y}{2}, y(0) = 1$ on $[0, 2]$ with $h = 1$ using the Modified-Euler's method. [6]
Hence compute the error if $y(x) = 3e^{-\frac{x}{2}} + x - 2$. [2]

(b) Apply Runge-Kutta method of order four (RK_4) to solve an initial-value problem $y' = -2xy^2, y(0) = 1$ from $x = 0$ to $x = 0.4$ using $h = 0.2$. [12]

END OF QUESTION PAPER

"Mathematics is the supreme judge; from its decisions there is no appeal"