GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS I

EMG 1101; EMN 1101

Examination Paper

NOVEMBER 2023

Time Allowed:	**3 hours**
Total Marks:	**100**
Examiner's Name:	**Mr. M. Mpofu**

<u>INSTRUCTIONS</u>

Candidates should answer **ALL** questions in Section A and attempt **ANY THREE** questions in Section B.

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks) Answer ALL questions from this section.

A1. (i) Solve $x^2 + 7x - 8 < 0$ [3]

(ii) Given $f(x) = x - 4$ and $g(x) = \begin{cases} \frac{x^2-16}{x+4} & x \neq -4 \\ k & x = -4 \end{cases}$

(a) find domain and range $f \circ g$ [3]

(b) Determine k so that $f(x) = g(x)$ for all x. [3]

A2. (i) Evaluate

(a) $\lim_{x\to 0} \left(\frac{\sqrt{x+3}-\sqrt{3}}{x} \right)$ [3]

(b) $\lim_{x\to +\infty} \frac{2x+5}{x^2-7x+3}$ [3]

(ii) Sketch the function

$$f(x) = \frac{3x+3}{x^2-3x-4}$$

[3]

A3. (i) Compute y'', if $x^3 - y^3 = 1$ [4]

(ii) Show that the reduction formula for evaluating the integral $I_n = \int_0^{\frac{\pi}{2}} \cos^n(x)\,dx$ is $\frac{n-1}{n} I_{n-2}$ [5]

Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^8(x)\,dx$ [2]

(iii) Find the area of the region sandwiched between the graphs of $f(x) = x^4 - x^2$ and $g(x) = 2x^2 - 2x^4$ [6]

A4. Use the ratio test to determine convergence of the series

$$\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2^n)!}$$

then find the radius and interval of convergence of the series. [5]

SECTION B (60 marks) Answer ANY THREE questions from this section.

B5. (i) Compute

(a) $\lim_{x \to 0^-} \frac{5}{2x}$ [3]

(b) $\lim_{x \to 0} \frac{\sin x}{x}$ [4]

(ii) (a) Use the $\epsilon - \delta$ definition to define a **limit**. [3]

(b) Hence, show that

$$\lim_{x \to 1}(5x - 3) = 2$$

[5]

(iii) The graph of the function $y = x^2 \sin \frac{1}{x}$ shows that the limit of y is 0 as x approaches 0.

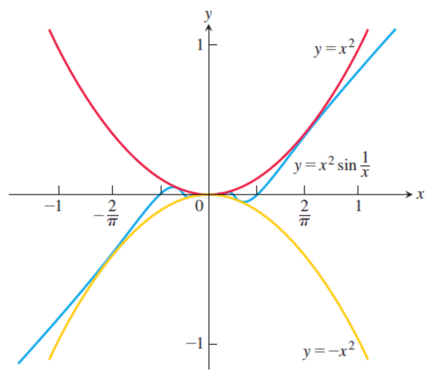

Figure 1:

Prove that

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$$

[5]

B6. (i) Define a **derivative** from first principle. [2]

(ii) A small funnel in the shape of a cone (Figure 3) is being emptied of fluid at the rate of 12 cubic centimeters per second. The height of the funnel is 20 centimeters and the radius of the top is 4 centimeters. How fast is the fluid level dropping when the level stands 5 centimeters above the vertex of the cone? (Hint:Volume of a cone is $\frac{1}{3}\pi r^2 h$.) [6]

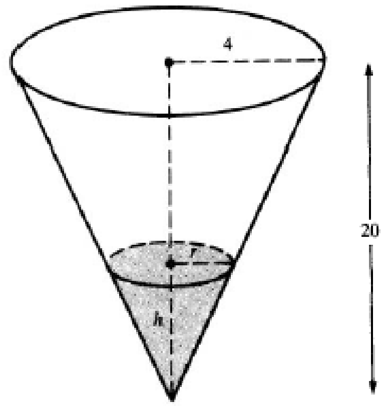

Figure 2:

(iii) Sketch the graph

$$f(x) = \frac{(x+2)^2}{x^2+2}$$

Determine all relative extrema, inflection points, asymptotes, concavity and suggest the behaviour at infinity. [12]

B7. (i) Evaluate

(a) $\int \frac{dx}{\sqrt{2^2-x^2}}$ [4]

(b) $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3} dx$ [5]

(c) $\int x^2 \sin 3x dx$ [5]

(ii) A hawk flying at 15 ms^{-1} at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground, where y is the height above the ground and x is the horizontal distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter. [6]

B8. (i) Find a formula for the nth term of the sequence

$$\sin\left(\frac{\sqrt{2}}{1+4}\right), \sin\left(\frac{\sqrt{3}}{1+9}\right), \sin\left(\frac{\sqrt{4}}{1+16}\right), \sin\left(\frac{\sqrt{5}}{1+25}\right), \cdots$$

[3]

(ii) (a) Find the Taylor series generated by

$$f(x) = \sin\left(3x + \frac{\pi}{2}\right)$$

at $x = \pi/4$. [5]

(b) Using the root test, determine if the series converges absolutely or diverges

$$\sum_{n=1}^{\infty} (-1)^n \left(1 - \frac{1}{n}\right)^{n^2}$$

[6]

(iii) Show that $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ diverges. [Hint: $2^{n^2} = (2^n)^n$] [6]

END OF QUESTION PAPER