GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

NUMERICAL METHODS AND ORDINARY DIFFERENTIAL EQUATIONS

EGS 2212

Examination Paper

MAY 2023

This examination paper consists of 3 printed pages

Time Allowed: **3 hours**

Total Marks: 100

Examiner's Name: Mr. R. G. Moyo

INSTRUCTIONS

Answer **ALL** questions in Section A and **ANY THREE** questions in Section B

ADDITIONAL REQUIREMENTS

Scientific calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Define the following terms as they are used in numerical methods:

(a) Interpolation [2]

(b) Runge Kutta Method of order 4 (RK4) [2]

(c) Upper triangular matrix [2]

A2. Use the Newton Raphson method to find $\sqrt{7}$ using $x_0 = 1.5$ giving your answer correct to 5 decimal places [5]

A3. (a) Determine the error term, $E_T(f,h)$ in the following numerical differentiation formula
$f''(x) = \frac{f(x+h)-2f(x)+f(x-h)}{h^2} + E_T(f,h)$ [3]

(b) Use Simpson's rule with $n = 4$ to obtain an approximation to $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin x}dx$ correct to 4 decimal places. [6]

A4. (a) Outline the steps followed when solving a non linear system using the Bisection Method. [4]

(b) Use the Bisection Method to find the root of $f(x) = cosx - \sqrt{x}$ lying between $[0,1]$. Use a tolerance of $\epsilon = 0.01$ and give your answer correct to 5 decimal places. [8]

(c) Solve the initial value problem(IVP) $y' = -2xy, y(1) = 2$ over $[1,3]$ using the Heun's method. Use $h = 1$. [8]

SECTION B (60 marks)

Answer ANY THREE questions from this section.

A5. (a) Discuss the applicability of interpolation and numerical differentiation in Geomatics and Surveying. [6]

(b) Construct a linear interpolation function given the 2 data-points $(3, 2)$ and $(5, 8)$ [2]

(c) Given that $f(x) = xe^x$, find approximations to $f''(0.5)$ using $h = 0.1$ and $h = 0.0001$. Compare the calculated values with the true value of $f''(0.5)$. Also comment on the effect of changing the step size. [12]

A6. (a) Determine the local truncation error (LTE) when solving an initial value problem using the Taylor's method. [4]

(b) Use Taylor's method of order $N = 3$ to solve $y' = 2x - y$ over $[0, 3]$ using $y(0) = 1$ and $h = 1$. [16]

A7. (a) Differentiate between precision and accuracy. [4]

(b) Solve the following system using the Naive Gaussian elimination. [16]

$$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$$
$$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$$
$$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$$
$$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$$

A8. (a) Construct the Newton polynomial function $P_3(x)$ for the points $(x; cosx)$ using $x = 0, 1, 2, 3$, correct to 6 decimal places. Use $P_3(x)$ to compute $\cos(2.5)$. Hence calculate the error for estimating the value of $cos(2.5)$ [8]

(b) Apply Runge-Kutta method of order four (RK_4) to solve an initial value problem $y' = -2xy^2, y(0) = 1$ from $x = 0$ to $x = 0.6$ with $h = 0.3$. [12]

END OF QUESTION PAPER

"Life is like Maths, if it goes too easy, something is wrong"