GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

MECHANICS (EGS 1209)

Final Examination Paper

June 2023

EPOCH MINE CAMPUS

Time Allowed: **3 hours**
Total Marks: **100**
Examiner's Name: **Mr. C.W Ndlovu**

INSTRUCTIONS

1. Answer **ALL** question in SECTION A.
2. Answer any **THREE** questions from SECTION B.
3. Use of calculators is permissible.

MARK ALLOCATION

Section A	**40**
Question A1	10
Question A2	10
Question A3	10
Question A4	10
Section B	**60**
Question B5	20
Question B6	20
Question B7	20
Question B8	20
Total Attainable	**100**

SECTION A ANSWER ALL QUESTIONS IN THIS SECTION (40 Marks)

Question A1

a) Using a well labelled table state the seven base quantities, and their SI Units **[7]**

b) Briefly describe the importance of studying classical mechanics to a Geomatics and Surveyor Student **[3]**

Question A2

a) State Newton's Three Laws of motion and briefly describe how it each is applicable to everyday life. **[6]**

b) Consider an object of mass m that is in free fall but experiencing air resistance. The magnitude of the drag force is given by,

$$F_{\text{drag}} = \frac{1}{2} C_D A \rho v^2$$

Where **ρ** is the density of air, ***A*** is the cross-sectional area of the object in a plane perpendicular to the motion, ***v*** velocity and C_D is the drag coefficient. Assume that the object is released from rest and very quickly attains speeds in which the above equation applies.

With the aid of simple and clear diagrams determine

(i) Show that the terminal velocity, is

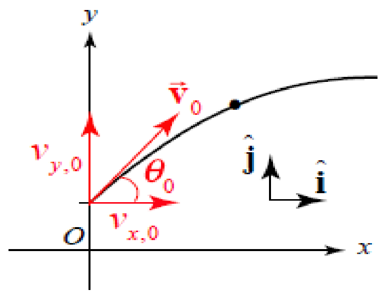
$$v_{\infty} = \sqrt{\frac{mg}{\beta}} = \sqrt{\frac{2mg}{C_D A \rho}}.$$ **[3]**

Hence calculate the magnitude of terminal velocity given that ***m=13,3kg*** C_D***= 0.87***

ρ= 1.225kg/m^3 [1]

Question A3

a) The diagram below shows the vector decomposition of the initial conditions of projectile motion



(Figure A3.0)

Using the diagram above and any other relevant classical mechanics assumptions derive the equation of the trajectory given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2} \qquad \text{(trajectory).}$$ **[6]**

b) Give two vectors

$$\vec{A} = 2\hat{i} + -3\hat{j} + 7\hat{k} \text{ and } \vec{B} = 5\hat{i} + \hat{j} + 2\hat{k},$$

Find

i. Sum of vector **A** and **B**
ii. Dot product of vector **A** and **B**
iii. Cross product of vector **A** and **B** **[4]**

Question A4

a) State Newton's Law of Gravitation **[2]**
b) Using the law stated above derive an equation for gravitational potential **[2]**
c) One of Kepler's laws of planetary motion relates the period and radius, state the Law and derive it
d) State the principle of conservation of energy **[1]**
e) State the principle of linear conservation of momentum **[1]**
f) One of the most important examples of periodic motion is simple harmonic motion (SHM), in which some physical quantity varies sinusoidal. Suppose a function of time has the form of a sine wave function,

$$y(t) = B\cos\left(\sqrt{\frac{k}{m}}\right)t$$

For the harmonic oscillator stated above derive an expression for

i. Velocity **[1]**
ii. Acceleration **[1]**
iii. Kinetic energy **[2]**

SECTION B (60 marks)

Answer ANY THREE questions from this section.

Question B5

a) For a body undergoing periodic motion (SHM), show that $x(t) = C\cos\omega_0 t + D\sin\omega_0 t = A\cos(\omega_0 t + \varphi)$, where $A = (C^2 + D^2)^{1/2} > 0$, and $\varphi = \tan^{-1}(-D / C)$. **[7]**

b) **Figure B5.0** shows a block *S* (the *sliding block*) with mass M =3.3 kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block *H* (the *hanging block*), with mass m =2.1 kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block *H* falls as the sliding block *S* accelerates to the right.

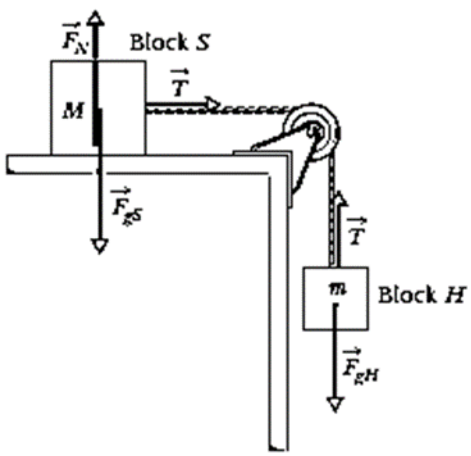


Figure B5.0

Find

i) the acceleration of block *S*, **[2]**

ii) the acceleration of block *H*, and **[2]**

iii) the tension in the cord. **[2]**

c) The earth, of mass $\mathbf{m_e = 5.97\times10^{24}\ kg}$ and (mean) radius $\mathbf{R_e = 6.38\times10^{6}\ m}$, moves in a nearly circular orbit of radius $\mathbf{r_{s,e} = 1.50\times10^{11}\ m}$ around the sun with a period T_{orbit} **= 365.25 days**, and spins about its axis in a period T_{spin} **= 23 hr 56 min**, the axis inclined to the normal to the plane of its orbit around the sun by 23.5° (in **Figure B5.1** the relative size of the earth and sun, and the radius and shape of the orbit are not representative of the actual quantities).

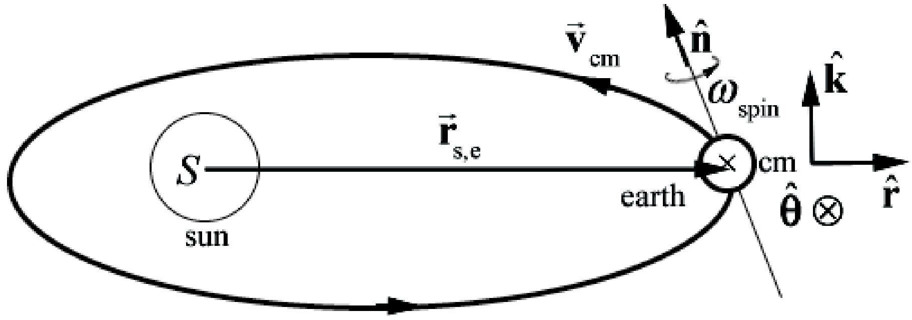


Figure B5.1

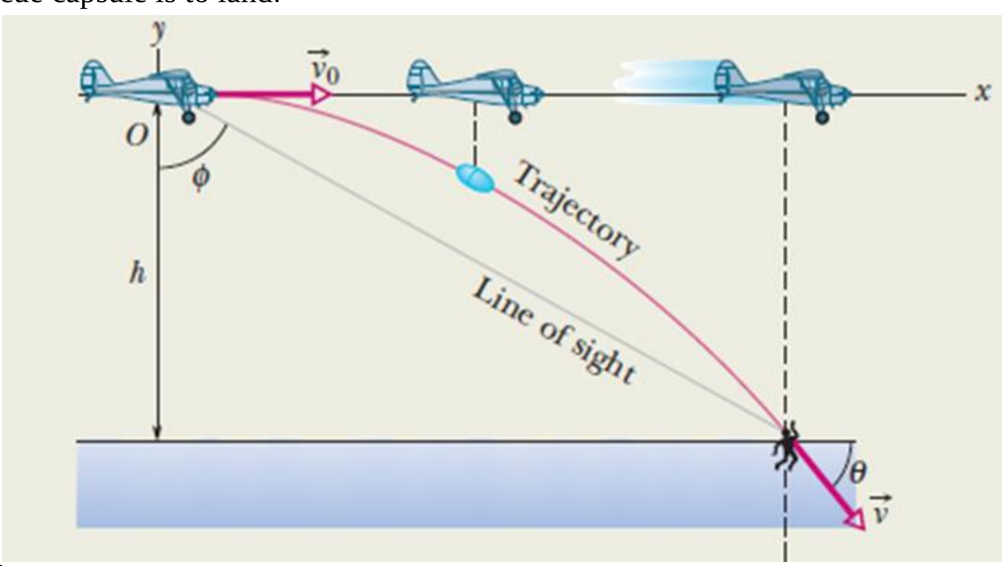
Find

i) The orbital angular momentum about S **[3]**

ii) The spin angular momentum is given by **[4]**

Question B6

a) The diagram shows a rescue plane to pick up victims of a boat disaster that was carrying surveyors coming from a survey camp in Madagascar, a rescue plane flies at 198 km/h (" 55.0 m/s) and constant height *h* =500 m toward a point directly over the victim, where a rescue capsule is to land.



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i) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

ii) As the capsule reaches the water, what is its velocity? **[6]**

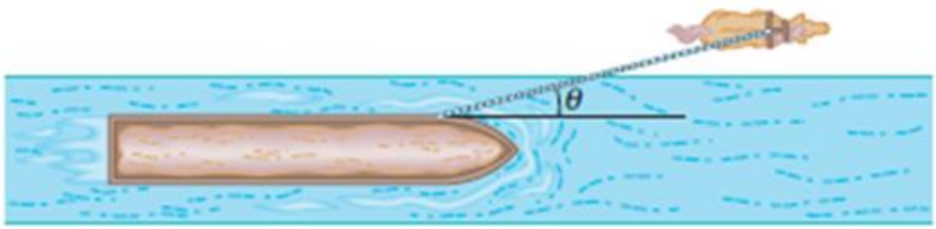
iii) For any projectile state the best angle of projection for maximum range **[1]**

iv) State Kepler's Laws of planetary motion**.** **[3]**

b) In earlier days, horses pulled barges down canals in the manner shown below (**Figure B6.0**). Suppose the horse pulls on the rope with a force of 7900N at an angle **ø=18** degrees to the direction of the motion of the barge, which is headed straight along the positive direction of an axis. The mass of the barge is 9500kg and the magnitude of its acceleration is 0,12m/s2

i) What is minimum force needed to move the barge with the given acceleration **[1]**

ii) Calculate the work done by the horse **[3]**



c) State the principle of conservation of energy **[2]**

A particle of mass $m = 2.0\ \text{kg}$ moves as shown in Figure 19.4 with a uniform velocity $\vec{\mathbf{v}} = 3.0\ \text{m}\cdot\text{s}^{-1}\ \hat{\mathbf{i}} + 3.0\ \text{m}\cdot\text{s}^{-1}\ \hat{\mathbf{j}}$. At time t, the particle passes through the point $(2.0\ \text{m}, 3.0\ \text{m})$.

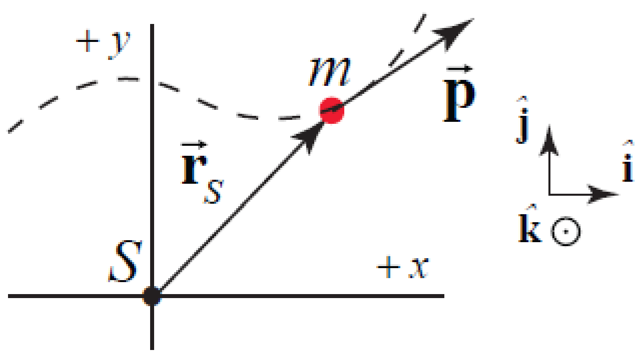


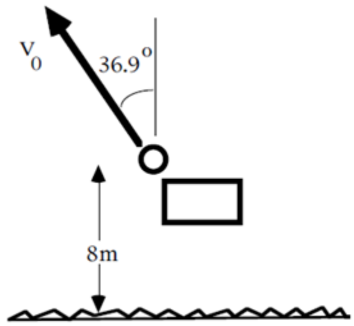
Figure 19.4

d) Find the direction and the magnitude of the angular momentum about the point *S* (the origin) at time *t* **[4]**

Question B7

a) State the Principle of Conservation of Angular Momentum **[2]**

b) A rock is thrown upward from a bridge at an initial height of 8 meters above the water at an initial speed of v0 and an angle of 36.9degrees from the vertical as shown. Use g=9,81m/s2 to solve this problem..



i) Write a set of equations for the horizontal and vertical positions and velocities of the rock as a function of time. Clearly indicate on your drawing your choice of axes and what point you are using as your origin. **[2]**

ii) The rock reaches its highest point in 2 seconds. How high is the rock above the water at that instant? (Hint: First you need to find v_0). **[4]**

c) A uniform rod of length l = 2.0 m and mass m = 4.0 kg is hinged to a wall at one end and suspended from the wall by a cable that is attached to the other end of the rod at an angle of $\beta = 30^o$ to the rod **(see Figure B7.0).** Assume the cable has zero mass. There is a contact force at the pivot on the rod. The magnitude and direction of this force is unknown. One of the most difficult parts of these types of problems is to introduce an angle for the pivot force and then solve for that angle if possible. In this problem you will solve for the magnitude of the tension in the cable and the direction and magnitude of the pivot force.

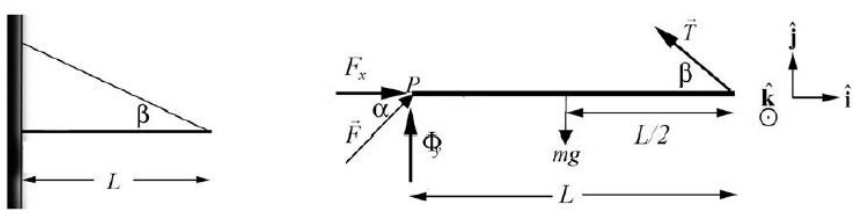


Figure B7.0

i) What is the tension in the cable? **[3]**

ii) What angle does the pivot force make with the beam? **[3]**

iii) What is the magnitude of the pivot force? **[3]**

iv) List any three methods of supporting a loaded beam **[3]**

Question B8

Suppose $x_1(t)$ and $x_2(t)$ are both solutions of the simple harmonic oscillator equation.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

By *ansatz* (educated guess) the linear combination $x(t) = x_1(t) + x_2(t)$ is also a solution of the SHO equation,

$x_1(t) = D\sin(\omega_0 t)$,

$x_2(t) = C\cos(\omega_0 t)$

a) Find the linear combination $x(t) = x_1(t) + x_2(t)$ **(1)**

b) Determine the velocity of the linear combination

   i. $v_{(x)}$ **(2)**

   ii. $a_{(x)}$ **(2)**

c) Show that the linear combination of the two solutions in also a solution to the simple harmonic oscillator equation. **(5)**

d) A block of mass m is attached to a spring with spring constant k and is free to slide along a horizontal frictionless surface. At $t = 0$, the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest, $v_{x,0} = 0$.

   i. What is the period of oscillation of the block?

   ii. What is the velocity of the block when it first comes back to the equilibrium position?

   iii) Show that the total energy of the system mid-way the equilibrium position and maximum displacement is, $\frac{1}{2}kx_0^2$ **(10)**

End of Question Paper.

