GWANDA STATE UNIVERSITY

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF GEOMATICS AND SURVEYING

CALCULUS

EGS 1207

This examination paper consists of 4 pages

Date:	**May 2023**
Total Marks:	**100**
Time:	**3 hours**
Examiner's Name:	**Mr. M. Mpofu**

INSTRUCTIONS

This paper consists of Section A (40 marks) and Section B (60 marks). Answer **ALL** questions in **Section A** and answer **ANY THREE** questions in **Section B**.

Use of calculator is permissible

ADDITIONAL MATERIALS

- Calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Given that $h(t) = \ln(t-3) + 1$,

(a) Find the domain and range of $h(t)$. [2]

(b) Sketch the graph $h(t)$. [4]

(c) Show that $(h \circ h^{-1})(t) = t$. [5]

A2. (a) Evaluate

(i) $lim_{y \to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$ [3]

(ii) $\int_0^\infty x^2 e^{-x} dx$ [4]

(b) If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an $R - ohm$ resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

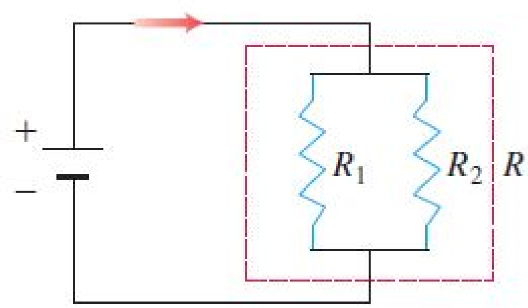

Figure 1: Parallel resistors

If R_1 is decreasing at the rate of 1 ohm/sec and R_2 is increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ $ohms$ and $R_2 = 50$ $ohms$ [4]

A3. Find $\frac{dy}{dx}$, given

(a) $y = \sqrt{\frac{x^2+x}{x^2}}$ [4]

(b) $x^3 + 4xy - 3y^{\frac{4}{3}}$ [4]

A4. (a) Find the Maclaurin series for $f(x) = \sin(3x)$ [5]

(b) Find the interval of convergence of the power series $\sum nx^n$ [5]

SECTION B (60 marks)

Answer ANY THREE questions from this section.

A5. (a) Given that

$$f(x) = \begin{cases} -x-2, & -4 \leq x \leq -1 \\ -1, & -1 < x \leq 1 \\ x-2, & 1 < x \leq 2 \end{cases}$$

Sketch the graphs of f and $f \circ f$ [5]

(b) Evaluate $\lim_{x\to\infty}\left(x - \sqrt{x^2+16}\right)$ [4]

(c) Using the $\epsilon - \delta$ definition, prove that

$$\lim_{x\to -4} \frac{16-x^2}{4+x} = 8$$

[4]

(i) Define continuity [3]

(ii) Consider

$$g(x) = \begin{cases} -2, & x \leq -1 \\ ax-b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

For what values of a and b is $g(x)$ continuous at every x? [4]

A6. (a) Define a derivative. [2]

(b) Water is leaking out of an inverted conical tank at a rate of $10,000cm^3/min$ at the same time that water is being pumped into the tank at a constant rate. The tank has height $6m$ and the diameter at the top is $4m$. If the water level is rising at a rate of $20cm/min$ when the height of water is $2m$, find the rate at which water is being pumped into the tank. [5]

(c) Sketch the graph of $f(x) = \frac{x^2-4}{2x}$ by identifying

(i) the domain of f and any symmetries the curve,

(ii) the derivatives,

(iii) the critical points of f,

(iv) where the curve is increasing and decreasing,

(v) the points of inflexion and/or the concavity of the curve,

(vi) any asymptotes and intercepts. [13]

A7. (a) Evaluate $lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$ [4]

(b) Find the area of the surface generated by revolving the curve $y = 5\sqrt{x}$, $1 \leq x \leq 2$, about the x-axis. [5]

(c) Given that $I = \int_0^1 \left(x^2 + 2e^{-3x}\right) dx$

(i) Using the Simpson rule (with four strips) evaluate the integral I [5]

(ii) Find the exact value of the integral I [5]

(iii) Find the absolute error. [1]

A8. (a) Distinguish between a **infinite series** and **sequence**. [4]

(b) (i) Use the ratio test to investigate the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$$

[7]

(ii) State the interval of convergence, if the function converges. [3]

(c) Show that $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$ diverges. [Hint: $2^{n^2} = (2^n)^n$] [6]

END OF QUESTION PAPER