GWANDA STATE UNIVERSITY

EMI/EMR: 1201

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHEMATICS II

EPOCH MINE CAMPUS

Mr M MPOFU

2021 EXAMINATIONS
Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

Instruments and Materials

- Calculator

SECTION A (40 marks)

Answer ALL questions from this section.

A1. Define the following terms

(a) Explicit solution and Implicit solution. [3]

(b) An nth-order Initial Value Problem (IVP). [3]

(c) Homogeneous linear nth-order ode. [4]

A2. Solve

(a) $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$ [3]

(b) $(5x + 4y)dx + (4x - 8y^3)dy = 0$ [3]

(c) $y'' - 2y' + 5y = 0$ [4]

A3. The population of a mining compound grows at a rate proportional to the population present at time t. The initial population of 50 increases by 15% in 5 years. What will be the population in 15 years? How fast is the population growing at $t = 15$? [5]

A4. (a) Find the general solution of the simultaneous pdes:

$$\frac{\partial V}{\partial t} = y^2$$

$$\frac{\partial V}{\partial y} = 2y + t$$

[5]

A5. (a) Find $L[f(t)]$

(i) $f(t) = (1 + e^{2t})^2$ [3]

(ii) $f(t) = cos5t + sin2t$ [3]

(b) Evaluate $L^{-1}\left(\frac{3s+1}{s^2-s-6}\right)$ [4]

SECTION B (60 marks)

Answer ANY THREE questions from this section.

A6. (a) Define a nonhomogeneous linear 2nd-order ode. [3]

(b) Using the method of undetermined coefficients, solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

[10]

(c) The radioactive isotope of Lead, $Pb - 209$, decays at a rate proportional to the amount present at time t and has a half-life of 3.3 hours. If 1 gram of this isotope is present initially, how long will it take for 90% of the Lead to decay? [7]

A7. Engineers and Mathematicians from Gwanda State University were studying the effect of diameter of wire rope used to hoist a 13 tonnes load out of a 609.6 m vertical shaft. They discovered that the ropes were vibrating forming some transverse waves along the rope, as shown in Figure 1.

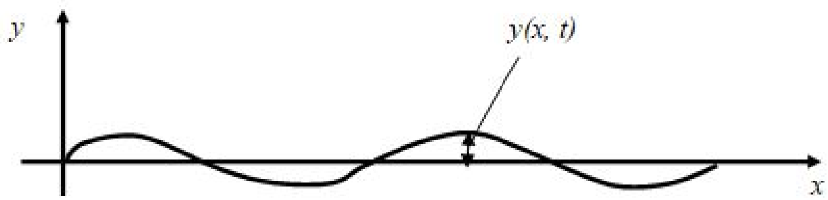

Figure 1: Transverse waves of a vibrating rope

$y(x,t)$ is the displacement of the string from equilibrium. Thus, to determine $y(x,t)$ at each point along the string at various times, a wave equation given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

where c is a constant-wave velocity, was used to model the scenario.

(a) Given
Initial conditions: $\frac{\partial y(x,0)}{\partial t} = 0$; $y(x,0) = f(x)$; $0 < x < l$ and
Boundary conditions: $y(0,t) = y(l,t) = 0$, $t > 0$,

show that

$$y(x,t) = \sum_{r=1}^{\infty} b_r sin\left(\frac{r\pi x}{l}\right) cos\left(\frac{r\pi ct}{l}\right)$$

where $r = 1, 2, \cdots$. [16]

(b) To determine the effect diameter, they modeled the relationship between the displacement $y(x,t)$ and the diameter D of the rope as

$$y \propto \frac{1}{D}$$

(i) Write down the equation connecting y and D, given that $y = 0.005$ when $D = 0.75$. [2]

(ii) Find y when $D = \frac{9}{8}$. [2]

A8. (a) Define Boundary Value Problem (BVP). [3]

(b) Solve

$$\frac{d^2i}{dt^2} + 10i = V(t)$$

where $V(t) = \begin{cases} 1 & -\pi < t \leq 0 \\ -1 & 0 < t \leq \pi \end{cases}$ and $V(t+2\pi) = V(t)$ [17]

A9. (a) State **three** types of Fourier transforms. [3]

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 0, & x \leq 0 \\ e^{-kx}cos(vx), & x > 0, k > 0 \end{cases}$$

[7]

(c) State **four** properties of Fourier transforms. [10]

A10. Solve the IVP using the Laplace transforms

(a) $\frac{dy}{dt} = e^{2t}$, $y(0) = \frac{1}{2}$ [8]

(b) $\frac{d^2y}{dt^2} + y = 1$, $y(0) = 2$, $y'(0) = 0$ [12]

END OF QUESTION PAPER

"*Do not worry about your difficulties in mathematics.*
I can assure you mine are still greater." Albert Einstein