# GWANDA STATE UNIVERSITY

EMR/EMI2101

FACULTY OF ENGINEERING AND ENVIRONMENT

**DEPARTMENT OF METALLURGICAL/MINING ENGINEERING**

EMR/EMI2101: ENGINEERING MATHEMATICS III

JANUARY 2019 EXAMINATION

Time : 3 hours

Question paper is approved In its original format 15/01/2019

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

**SECTION A: Answer ALL questions in this section [40].**

**A1.** Find $\int_c \mathbf{F}.\, d\mathbf{c}$ for $F(x,y) = -y\hat{\imath} + x\hat{\jmath}$ where $c = \cos t\ \hat{\imath} + 2\sin t\ \hat{\jmath},\ 0 \leq t \leq \frac{\pi}{2}$. [5]

**A2.** (a) Evaluate the integral $\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{5} x\ dzdydx$. [5]

(b) Sketch the region R in the $xy$ plane bounded by the curves $y^2 = 2x$ and $y = x$, and find its area. [8]

**A3.** Compute $\int_c \mathbf{F}.\, d\mathbf{r}$ for $F(x,y) = \hat{\imath} - y\hat{\jmath}$ where C is part of a circle of radius 3 in the first quadrant from $(3,0)$ to $(0,3)$. [5]

**A4.** (a) By finding the Fourier series the square wave

$$f(x) = \begin{cases} +1, & \text{if } \ 0 \leq x \leq \pi \\ 0, & \text{if } \ -\pi \leq x \leq 0 \end{cases}$$

$f(x + 2\pi) = f(x)$, find an expression for $\pi$. [5]

**A5.** (a) Let D be the region bounded by the unit cube defined by $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$. Verify the divergence theorem if $F = xy\hat{i} + yz\hat{j} + xz\hat{k}$. [6]

(b) State the Green's theorem. [2]

(c) Show that the force $\mathbf{F} = (4xy + 3x^2z^2)\widehat{i} + 2x^2\widehat{j} + 2x^3z\widehat{k}$ is a conservative force field. [4]

**SECTION B: Answer THREE questions in this section [60].**

**B6.** (a) Find the volume of the region B bounded by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the plane $z = b$, where $a > b > 0$. [8]

(b) Evaluate the integral $\displaystyle\int_0^1 \int_{\sqrt{3}y}^{\sqrt{4-y^2}} \sqrt{x^2 + y^2}\, dx\, dy.$ [5]

(c) Find the mass of the 3D region B given by $x^2 + y^2 + z^2 \leq 4$, $x \geq 0$, $y \geq 0$, $z \geq 0$, if the density is equal to $xyz$. [7]

**B7.** (a) Sketch the graph $f(x)$, in the interval $-2\pi \leq x \leq 2\pi$ where

$$f(x) = \begin{cases} x, & \text{if } \; 0 \leq x \leq \pi \\ \pi, & \text{if } \; \pi \leq x \leq 2\pi \end{cases}$$

and has a period of $2\pi$ [2]

(b) Derive the coefficients $a_0, a_r, b_r$, $(r = 1, 2, 3, .......)$ for a general Fourier series. [10]

(c) By finding the Fourier series of $f(x) = |x|$, $-2 \leq x \leq 2$, $f(x+4) = f(x)$ show that $\pi^2 = 8\left(1 + \frac{1}{9} + \frac{1}{25} \cdots\right)$ [8]

**B8.** (a) Compute the line integral $\displaystyle\int_C \mathbf{F}.d\mathbf{s}$ for the path $c(t) = (t^2, t^3, t)$ with $0 \leq t \leq 1$ and vector field $F(x, y, z) = x\hat{i} + z\hat{j} + x\hat{k}$. [5]

(b) Compute the line integral $\displaystyle\int_c zdx + ydy + xdz$ for the path $c(t) = (e^{t^2}, \ln(t + 1), \cos(t))$ with $0 \leq t \leq 1$. [5]

(c) Evaluate $\displaystyle\int\int_R (x - y)\sqrt{(x + y)}dA$ where R is the parallelogram with vertices $(0, 0)$, $(-1, 1)$, $(2, 4)$ and $(3, 3)$ by using the change of variables $u = x - y$ and $v = x + y$. [10]

**B9.** (a) Verify the divergence theorem over $S$ for $\mathbf{F} = x\hat{i} + y\hat{j} + (z-1)^2\hat{k}$, where $S$ is the region bounded by the hemisphere $x^2 + y^2 + (z-1) = 9$ in the plane $z = 1$. [7]

(b) Use Green's theorem to evaluate the following integral
$\oint_C (x^4 - 2y^3)dx + (2x^3 - y^4)dy$ where $C$ is circle $x^2 + y^2 = 4$. [6]

(c) Verify Stoke's theorem for $\mathbf{F} = (2x - y)\hat{i} + yz^2\hat{j} + y^2z\hat{k}$ over S, where $S$ is the upper half plane of the sphere $x^2 + y^2 + z^2 = 1$. [7]

**END OF QUESTION PAPER**