FACULTY OF ENGINEERING AND ENVIRONMENT

DEPARTMENT OF METALLURGICAL/MINING ENGINEERING

ENGINEERING MATHEMATICS

EMR/EMI 1101

Final Examination Paper

January 2019

This examination paper consists of 4 pages

Time Allowed: 3 hours

Total Marks: 100

Examiner's Name: Mr T GWEBU

INSTRUCTIONS

1. Answer **ALL QUESTIONS IN SECTION A**
2. Answer **ANY THREE QUESTIONS FROM SECTION B**
3. Use of calculators is permissible

MARK ALLOCATION

SECTION A	40 MARKS
SECTION B	60 MARKS
Total Attainable	100 MARKS

Question Paper is approved as It is

15/01/2019

SECTION A

A1. Find the limits

(a) $\lim_{x \to -5} \frac{x^2 - 25}{x^2 + 2x - 15}$, [3]

(b) $\lim_{x \to \infty} \frac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4}$, [3]

(c) $\lim_{x \to 0} \left(\frac{1}{x^2} \right)^x$. [4]

A2. Determine if the following function is continuous or discontinuous at $x = 6$.

$$f(x) = \begin{cases} 2x, & x < 6 \\ x - 1, & x \geq 6. \end{cases}$$ [4]

A3. (a) Let $f(x) = \sqrt{2x - 1}$. Evaluate $f'(5)$ from first principles. [4]

(b) Find the first derivative of $y = (x^2 + 4)^{2x}$. [4]

A4. Evaluate the following

(a) $\int 90x^2 \sin(2 + 6x^3)dx$, by using a substitution, [3]

(b) $\int \frac{1}{1 - x^2} dx$, by the method of partial fractions. [4]

A5. (a) Determine the modulus and argument of the complex number $z = 2 + 3i$ and express z in polar form. [5]

(b) Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$. [6]

SECTION B

B6. (a) Find the reduction formula for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$. Hence, evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x dx$. [8]

(b) Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$. [6]

(c) Compute the volume of the solid generated by revolving about y-axis, the region enclosed by the parabolas $y = x^2$ and $8x = y^2$. Use the method of washers. [6]

B7. (a) Differentiate $y = e^{x^2} sech^{-1} x$. [5]

(b) Find the coordinates of any stationary points on the curve $y = x^3 - 12x - 5$ and distinguish between them. Hence sketch the curve. [8]

(c) We want to construct a window whose middle is a rectangle and the top and bottom of the window are semi-circles. If we have $50m$ of framing material, what are the dimensions of the window that will let in the most light? [7]

B8. (a) Find all the asymptotes of the curve $y = \dfrac{x^3}{x^2 + x - 2}$. [4]

(b) Use De Moivre's theorem to express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$. [4]

(c) Find the square roots of $z = 5 + 3i$ in rectangular form correct to 4 significant figures. [6]

(d) By considering the real and imaginary parts, evaluate

$$\int e^{4x} \cos 5x dx.$$

[6]

B9. (a) Find the volume of the parallelopiped with adjacent edges PQ, PR and PS. $P(-2, 1, 0)$, $Q(2, 3, 2)$, $R(1, 4, -1)$ and $S(3, 6, 1)$. [6]

(b) Find vectors **v** and **w** such that **v** is parallel to $(1, 2, 3)$, $\mathbf{v}+\mathbf{w} = (7, 3, 5)$ and **w** is orthogonal to $(1, 2, 3)$. [5]

(c) Find the distance between the skew lines

$$L_1 : x = 1 + t, y = -2 + 3t, z = 4 - t,$$

$$L_2 : x = 2s, y = 3 + s, z = -3 + 4s.$$

[9]

END OF QUESTION PAPER