

# GWANDA STATE UNIVERSITY 

EMI/EMR: 2201

FACULTY OF ENGINEERING AND THE ENVIRONMENT DEPARTMENTS OF MINING AND METALLURGY

APPLIED MATHEMATICS

## EPOCH MINE CAMPUS

Mr R.G. MOYO

2021 EXAMINATIONS
Time: 3 hours

Candidates should attempt ALL questions from Section A and ANY THREE questions from Section B.

Instruments and Materials

- Calculator.
- Graph paper
- Statistical Tables


## SECTION A (40 marks)

## Answer ALL questions from this section.

A1. Define the following terms
(a) Interpolation
(b) Numerical differentiation
(c) Probability
(d) Truncation error
(e) Hypothesis testing

A2. (a) Construct a linear interpolation function given the 2 data points $(3,2)$ and $(5,8)[3]$
(b) Given the table below

| $x$ | 1.3 | 1.4 | 1.45 | 1.5 | 1.55 | 1.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 4.77 | 5.677 | 6.182 | 6.732 | 7.3028 | 7.9248 |

Evaluate
(i) $f^{\prime}(1.5)$ using the backward difference method
(ii) $f^{\prime}(1.4)$ using the central diference method
(iii) $f^{\prime \prime}(1.5)$

A3. Mr. R. G. delivers an average of two lectures per week at GSU. Assuming that the number of lectures delivered can be modelled by a Poisson distribution, find the probability that he conducts
(a) exactly three lectures in a given week
(b) more than four lectures in a given week
(c) exactly four lectures in a given fortnight
(d) no lectures on a given day, assuming that GSU operates on a five-day week.

A4. (a) Use Simpson's rule with $n=2$ to obtain an approximation to $\int_{0}^{\frac{\pi}{4}} x \cos x d x$
(b) Solve $y^{\prime}=\frac{x-y}{2}$ on $[0,3]$ with $y(0)=1$ and $h=1$ using the Euler's method. Hence calculate the error if $y(x)=3 e^{-\frac{x}{x}}+x-2$

## SECTION B (60 marks)

## Answer ANY THREE questions from this section.

A5. (a) Determine the local truncation error (LTE) when solving an initial value problem using the Taylor's method
(b) Use Taylor's method of order $N=3$ to solve $y^{\prime}=2 x-y$ over [0,3] using $y(0)=1$ and $h=1$

A6. (a) During the 2014 World cup in a certain University, the probability that there was electricity on any particular day was $\frac{1}{3}$. In case that there was no electricity, a generator would be switched on. Independently, the probability that Welton watched a soccer match being screened live was $\frac{1}{4}$.
(i) Represent the above information by a means of a tree diagram.
(ii) Find the probability that there was no electricity and Welton did not watch the match being screened live.
(b) 'Ntozonke Mine' needs to purchase cars for its top managers. The table below displays data on age and price for a sample of eleven cars being sold by Toyota car company. Ages are in years while prices are in thousand dollars.

| Age | 5 | 4 | 6 | 5 | 5 | 5 | 6 | 6 | 2 | 7 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Price | 8.5 | 10.3 | 7 | 8.2 | 8.9 | 9.8 | 6.6 | 9.5 | 16.9 | 7 | 4.8 |

(i) Draw a scatter diagram for price against age
(ii) Calculate the equation of the regression line of price on age of the car
(iii) Draw the line of the equation on a scatter diagram in (i) and use it to estimate the price o a 3 year old car.
(iv) Find the product-moment correlation co-efficient and comment on it

A7. (a) Define the following terms
(i) Runge-Kutta method of order four $\left(R K_{4}\right)$
(ii) First order ordinary differential equation
(b) Apply Runge-Kutta method of order four $\left(R K_{4}\right)$ to solve an initial value problem $y^{\prime}=-2 x y^{2}, y(0)=1$ from $x=0$ to $x=0.9$ with $h=0.3$

A8. (a) Outline the steps followed when carrying out a $\chi^{2}$ test.
(b) The following data shows the ownership of satellite dishes by different social classes in a randomly chosen sample of 150 households at Gwanda State University.

| Social class | Number of people who <br> own satellite dishes | Number of those without <br> a satellite dish |
| :--- | :--- | :--- |
| Executive staff | 15 | 10 |
| Academic Staff | 23 | 8 |
| Students | 54 | 40 |

Test at $5 \%$ level of significance to establish if there is an association between ownership of a satellite dish and social class at GSU.
[15]

A9. (a) Given that $f(x)=x e^{x}$, use a three-point formula with $h=0.1$ and $h=0.001$ to find approximations to $f^{\prime \prime}(2.5)$. Compare the calculated values with the true value of $f^{\prime \prime}(2.5)$. Comment on the effect of changing your step size.
(b) Discuss the applicability of interpolation and numerical differentiation in a mineral processing plant.

## END OF QUESTION PAPER

"Do not worry about your difficulties in mathematics.
I can assure you mine are still greater." Albert Einstein

