GWANDA STATE UNIVERSITY

EMI/EMR: 1201

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENTS OF MINING AND METALLURGY

ENGINEERING MATHS II

EPOCH MINE CAMPUS

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SECOND SEMESTER 2021: EXAMINATION
Time : 3 hours

Candidates should attempt **ANY FIVE** questions from this paper with seven questions (20 marks each).

Instruments and Materials

- Calculator.

A1. (a) Find the general solutions of the following ODEs.

(i) $y'' - 2y' + y = 0$. [4]

(ii) $y'' + 3y' - 4y = 0$. [4]

(b) Find particular solutions of the following ODEs.

(i) $y'' - y' + 3y = \sin t$. [6]

(ii) $y'' + 2y' - 3y = e^t$. [6]

A2. (a) Suppose that the coefficient functions $p(t)$, $q(t)$ are continuous in the interval $0 < t < \pi$, and the functions $y_1(t) = t$, $y_2(t) = \sin t$ are solutions of the ODE

$$y^{('')} + p(t)y^{(')} + q(t)y = 0$$

for $0 < t < \pi$:

(i) Compute the Wronskian of y_1, y_2. [3]

(ii) Are they linearly independent on the interval $0 < t < \pi$? [2]

(iii) Is the pair $\{y_1; y_2\}$ a fundamental set of solutions for the ODE? [2]

(iv) Could $p(t)$, $q(t)$ be continuous on $\pi < t < \pi$? Explain your answers. [3]

(b) Hence, find the solution $y(t)$ of the initial value problem for the ODE with initial conditions

$$y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = 2$$

[10]

A3. (a) Define a matrix. [2]

(b) Convert the differential equation $\dddot{x} - 2\ddot{x} + \dot{x} = 0$ into the matrix equation $\mathbf{\dot{x}(t) = A(t)x(t) + f(t)}$ [5]

(c) If

$$\mathbf{A} = \begin{pmatrix} 5 & 7 & 0 & 0 \\ -3 & -5 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

(i) Find the characteristic equation and eigenvalues of $\mathbf{A}$. [5]

(ii) State one use of the eigenvalues. [1]

(d) $y^{'''} - 6y^{''} + 11y^{'} - 6y = 0$, $y^{'}(\pi) = 0$, $y^{''}(\pi) = 1$ [7]

A4. (a) The displacement $y(t)$ of an undamped oscillator of mass $m > 0$ on a spring with spring constant $k > 0$, and initial displacement a $a \neq 0$ and initial velocity 0 satisfies

$$my^{''} + ky = 0$$

for $y(0) = a$, $y^{'}(0) = 0$.

(i) Solve this initial value problem. [10]

(ii) Show that the solution is periodic with period T , meaning that $y(t + T) = y(t)$, and express T in terms of m and k. [6]

(iii) For what times t does the oscillator pass through equilibrium, meaning that $y(t) = 0$? [4]

A5. (a) Define a partial differential equation. [2]

(b) How different is partial differential equation and ordinary differential equation. [2]

(c) The following equation is used in modelling of compressible fluid flow

$$\frac{\partial p}{\partial t} + div(\rho \mathbf{v}) = 0$$

(i) What is the name given to this equation? [2]

(ii) State the condition of steady flow. [2]

(iii) State the condition of incompressibility. [2]

(d) Obtain the linearized system of the nonlinear dynamical system below

$$\frac{dx}{dt} = 2x - xy$$
$$\frac{dy}{dt} = 2y - xy$$

[10]

A6. (a) What is a Differential Equation? [2]

(b) (i) Find the solution of the initial value problem

$$ty^{'} = \frac{1}{y+1}$$

for $y(1) = 0$ [6]

(ii) For what t-interval is the solution defined? [2]

(c) Suppose that a is a constant, and consider the initial value problem

$$y^{'} - y = e^{at}$$

for $y(0) = 0$

(i) Find the solution if $a \neq 1$. [5]

(ii) Find the solution if $a = 1$ [5]

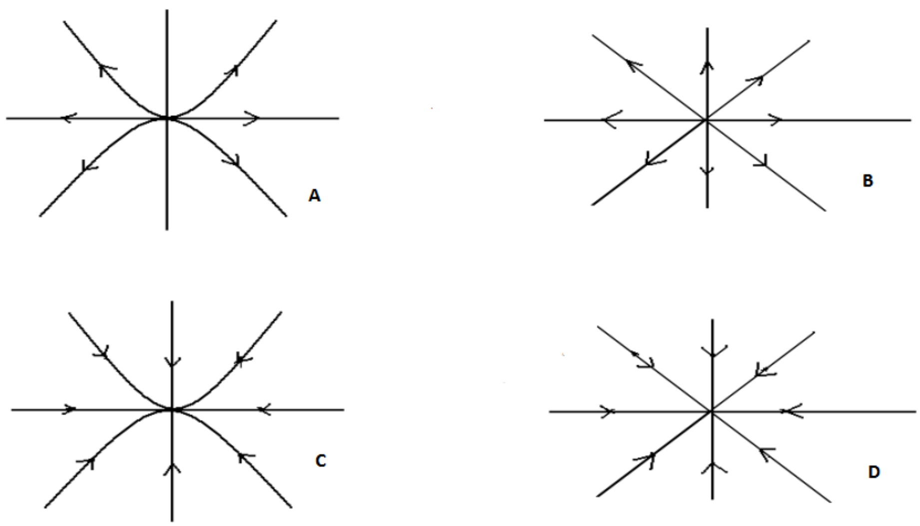
A7. (a) Define the following terms

(i) Fixed Point [2]

(ii) steady State [2]

(iii) Phase Plane [2]

(b) Some examples of Phase diagrams in two-dimension will appear as:



Complete the following table for stability analysis

Stable Phase Plane	Unstable Phase Plane

[6]

(c) Analyse the stability of equilibrium point for the system

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = -x + y$$

[8]

END OF QUESTION PAPER

"*Do not worry about your difficulties in mathematics.*
I can assure you mine are still greater." Albert Einstein