GWANDA STATE UNIVERSITY

EGS1204 &EGS1207

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

CALCULUS FOR GEOMATICS AND CALCULUS

EPOCH MINE CAMPUS

Mr M NDLOVU

SECOND SEMESTER 2021: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

Instruments and Materials

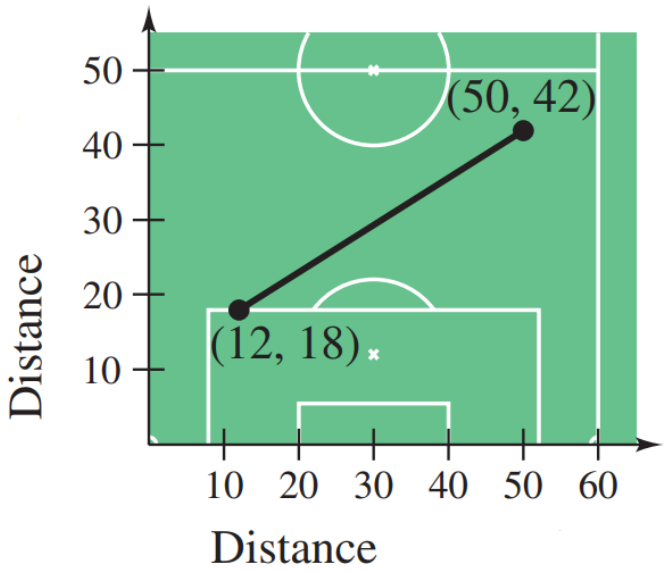
- Calculator.

SECTION A: Answer ALL questions [40].

A1. (a) True or False Differentiation and Integration are the two branches of calculus [2]

(b) Solve $|5 - x| \leq -2$ [5]

(c) A soccer player passes the ball from a point that is 18 metres from the endline and 12 metres from the sideline. The pass is received by a teammate who is 42 metres from the same endline and 50 metres from the same sideline, as shown in the figure. How long is the pass? [3]



A2. (a) Evaluate this integral $\int x^n dx$ for

(i) $n < 0$ [2]

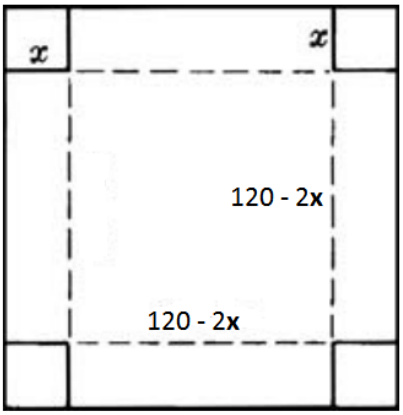
(ii) $n > 1$ [2]

(iii) $n = 1$ [2]

(b) Hence, solve $\int \left(1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx$ [4]

A3. (a) Define a function. [2]

(b) From each corner of a square of tin, 120 cm on a side, small squares of side x (in cm) are removed, and the edges are turned up to form an open box.



(i) Express the volume V of the box (in cubic cm) as a function of x, [4]

(ii) Determine the domain of the function. [2]

(iii) Compute the maximum volume of box. [4]

A4. (a) Compute $\frac{dy}{dx}$ given $y = x^5 + 5x^4 - 10x^2 + 6$ [4]

(b) Investigate the successive derivatives of $f(x) = x^{\frac{4}{3}}$ at $x = 0$.
Hint: successive derivates can be $f^{'}(0)$ and $f^{''}(0)$ [4]

SECTION B: Answer ANY three questions [60].

B5. (a) Find the interval of convergence of the given power series

(i) $\sum nx^n$ [4]

(ii) $\sum f(x)$ for $f(x) = \dfrac{(x-4)^n}{n^2}$ [5]

(b) Find Maclaurin series for $f(x) = \ln(1+x)$. [5]

(c) Determine whether the solutions function $z = x^2 - y^2$ satisfies Laplaces equation given by $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ [6]

B6. (a) Sketch the curve with equation

$$y = x^3 - 5x^2 - 8x + 12$$

Identifying

(i) where will it cut the x-axis? [1]

(ii) where will it cut the y-axis? [4]

(iii) where is it stationary? [3]

(iv) where it is increasing/decreasing especially in the neighbourhood of the stationary points. [3]

(v) how it behave as $x = \pm\infty$ [2]

(b) Using the standard laws for limits, show that

$$\lim_{(x;y)\mapsto(3;1)} \left(\frac{3xy^2}{7+y} + \frac{1}{2}xy \right) = \frac{21}{8}$$

. [3]

(c) Given that $z = x^2 + 3xy + y^2$ show that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

[4]

B7. (a) An environmental study of a certain community in Tsholotsho suggests that the average daily level of land cover in rangelands (Pastures) will be $Q(p) = \sqrt{-0.5p + 39}\ m^2$ when the livestock population is p thousand. It is estimated that t years from now, the livestock population will be $p(t) = 8 + 0.2t^2$ thousand.

(i) Express the level of land cover in rangelands as a function of time. [2]

(ii) What will the land cover be 3 years from now? [4]

(iii) When will the land cover levels drop to reach less 5 m^2? [4]

(b) The radius of curvature, ρ, of part of a surface when determining the surface tension of a liquid is given by: $\rho = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ Find the radius of curvature (correct to 4 significant figures) of the part of the surface having parametric equations

(i) $x = 3t$, $y = \dfrac{3}{t}$ at the point $t = \dfrac{1}{2}$. [10]

B8. (a) Show that the differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$ is satisfied when $y = xe^{2x}$ [5]

(b) If $I_n = \displaystyle\int_0^a x^n e^{-x} dx$ show that $I_n - (n+2)I_{n-1} + a(n-1)I_{n-2} = 0$ [7]

(c) Evaluate the double integral of

$$\int_1^2 \int_y^{3y} (x + y) dx dy$$

[8]

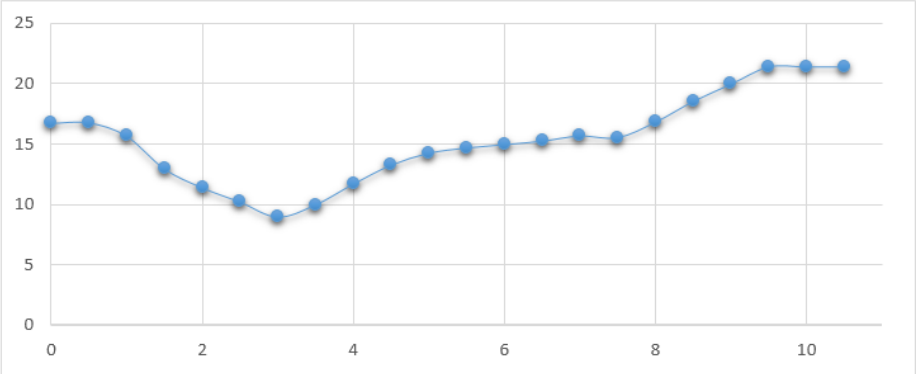
B9. In this study we are going to apply curve fitting technique and technological software to elaborate the application of mathematics in real world sceneries around Gwanda State University. Curve fitting is the process of constructing a curve, or mathematical function, that has the best fit to a series of data points, possibly subject to constraints. Comment on the following stages of this study presentated by these graphs.

(a) Stage 1: Data collection [3]

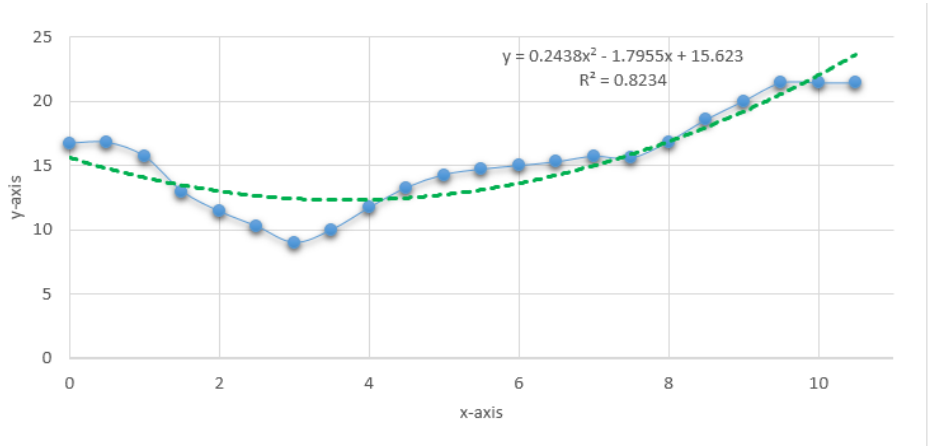
(b) Stage 2: Data cleaning or data transformation [3]

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5
y	16.75	16.8	15.71	12.99	11.42	10.25	8.99	9.99	11.71	13.28	14.28	14.71	14.99	15.28	15.71	15.57	16.85	18.57	20	21.43	21.43	21.43
y top down	3.25	3.2	4.29	7.01	8.58	9.75	11.01	10.01	8.29	6.72	5.72	5.29	5.01	4.72	4.29	4.43	3.15	1.43	0	-1.43	-1.43	-1.43
y down up	16.75	16.8	15.71	12.99	11.42	10.25	8.99	9.99	11.71	13.28	14.28	14.71	14.99	15.28	15.71	15.57	16.85	18.57	20	21.43	21.43	21.43

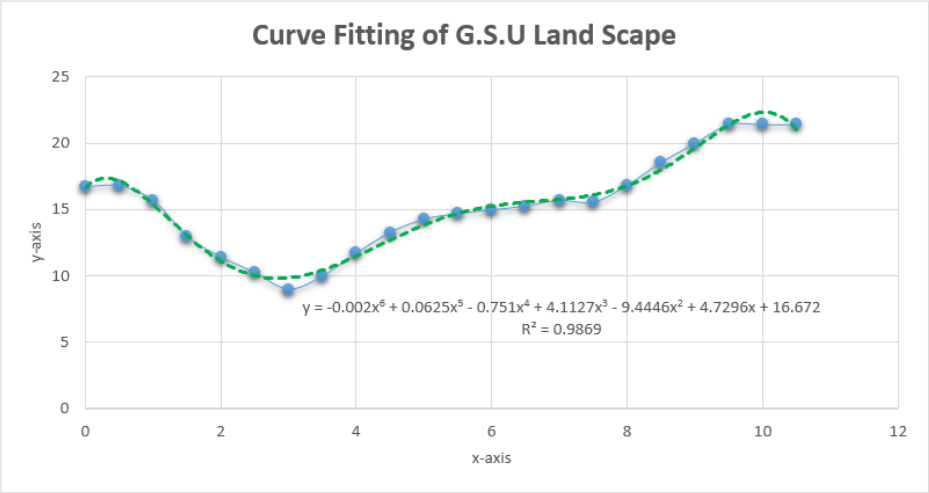
(c) Stage 3: Scatter Plot [2]



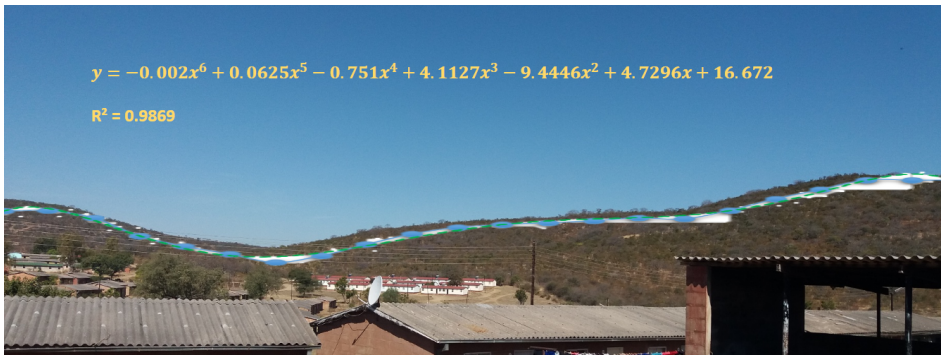
(d) Stage 4: Curve fitting (i) [3]



(e) Stage 5: Curve fitting (ii) [3]



(f) Stage 6: Stage Compare different fitted curves and conclusion [4]



END OF QUESTION PAPER

David Hilbert

"*Mathematics is a game played according to certain simple rules with meaningless marks on paper.*"