GWANDA STATE UNIVERSITY

EGS: 1102

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF GEOMATICS AND SURVEYING

ENGINEERING MATHEMATICS

EPOCH MINE CAMPUS: FILABUSI

Mr M NDLOVU

ACADEMIC YEAR 2021: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions in SECTION A and **Any** THREE questions from SECTION B.

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

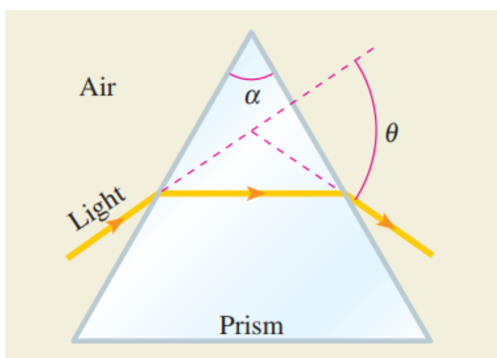
A1. (a) The time for one complete cycle of a point in simple harmonic motion is its .... [2]

(b) The number of cycles per second of a point in simple harmonic motion is its .... [2]

(c) The sun is 20° above the horizon. Find the length of a shadow cast by a park statue that is 12 m tall. [4]

(d) The index of refraction of a transparent material is the ratio of the speed of light in a vacuum to the speed of light in the material. Some common materials and their indices are air (1.00), water (1.33), and glass (1.50). Triangular prisms are often used to measure the index of refraction based on the formula

$$n = \frac{sin\left(\frac{\theta}{2} + \frac{\alpha}{2}\right)}{sin\left(\frac{\theta}{2}\right)}$$



(i) Write the index of refraction as a function of $\cot\frac{\theta}{2}$. [6]

(ii) Find θ for a prism made of glass. [6]

A2. (a) Find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph using the asymptotes as an aid
$\frac{(x-5)^2}{36} - \frac{(y+3)^2}{16} = 1$ [6]

(b) Find the exact solutions of the equation in the interval $[0, 2\pi)$
$(\sin 2x + \cos 2x)^2 = 0$ [6]

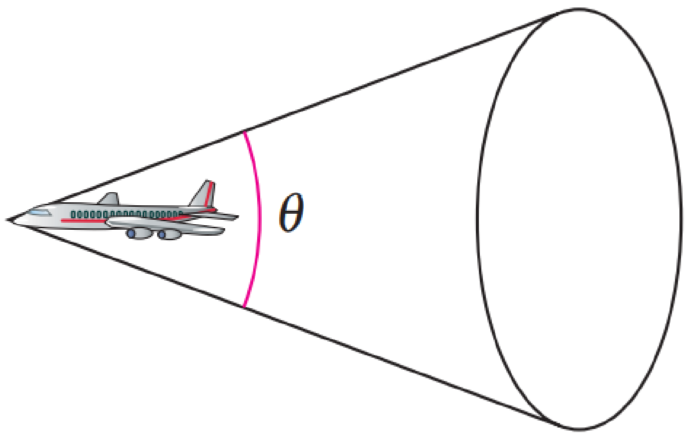
(c) Solve the system of equations

$$\begin{aligned} 2x + y + 2z &= 4 \\ 2x + 2y &= 5 \\ 2x - y + 6z &= 2 \end{aligned}$$

[8]

SECTION B: Answer ANY three questions [60].

B3. (a) The mach number M of an airplane is the ratio of its speed to the speed of sound. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. The mach number is related to the apex angle θ of the cone by $\sin\left(\frac{\pi}{2}\right) = \frac{1}{M}$



(i) Find the angle θ that corresponds to a mach number of 1. [2]

(ii) Find the angle that corresponds to a mach number of 4.5. [2]

(iii) The speed of sound is about 760 miles per hour.
Determine the speed of an object with the mach numbers from parts (i) and (ii). [6]

(b) Use the fundamental trigonometric identities to simplify

$$\frac{\sin\theta - 1}{\cos\theta} - \frac{\cos\theta}{\sin\theta - 1}$$

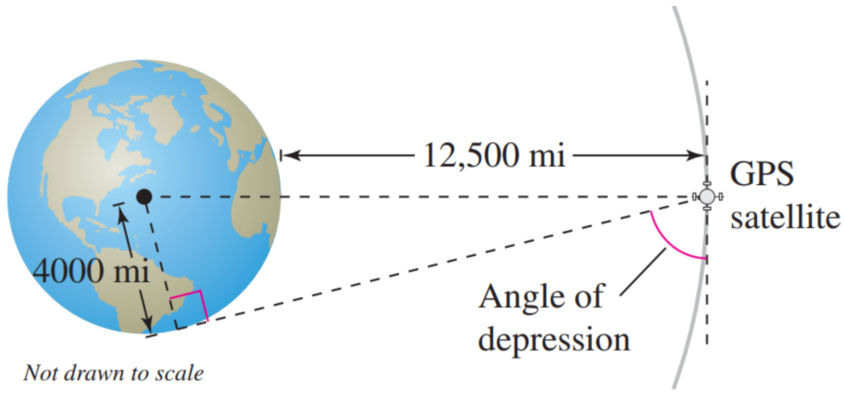
[5]

(c) Find the angle α between two non vertical lines L_1 and L_2

$$L_1 : 3x - 2y = 5$$
$$L_2 : x + y = 1$$

[5]

B4. (a) A Global Positioning System satellite orbits 12,500 miles above Earths surface (see figure). Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 metres. [5]



(b) A popular theory that attempts to explain the ups and downs of everyday life states that each of us has three cycles, called biorhythms, which begin at birth. These three cycles can be modeled by sine waves.

Physical (23 days): $P = \sin \dfrac{2\pi t}{23}$, $t \geq 0$

Emotional (28 days): $P = \sin \dfrac{2\pi t}{28}$, $t \geq 0$

Intellectual (33 days): $P = \sin \dfrac{2\pi t}{33}$, $t \geq 0$

where is the number of days since birth. Consider a person who was born on July 20, 1998

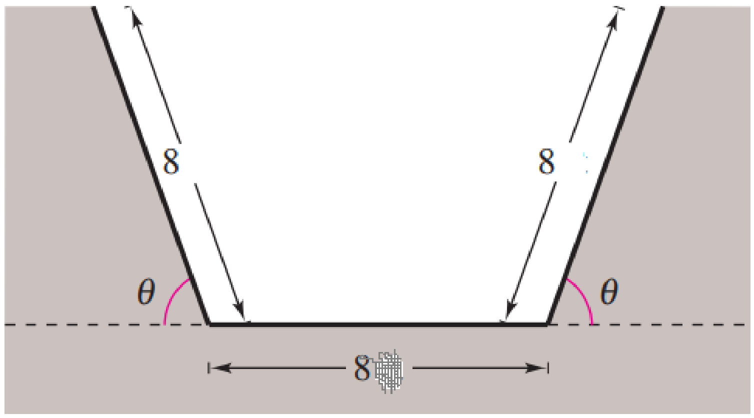
(i) Graph the Physical model in the same viewing window for $7300 \leq t \leq 7360$ [6]

(ii) Describe the persons Physical model during the month of September 2018. [3]

(iii) Calculate the persons three energy levels on September 22, 2018. [6]

B5. (a) Prove that $\cos x(\tan^2 x + 1) = \sec x$ [5]

(b) The cross section of an irrigation canal is an isosceles trapezoid of which 3 of the sides are 8 m long. The objective is to find the angle θ that maximizes the area of the cross section.[*Hint:* the area of the trapezoid is $\frac{h}{2}(b_1 + b_2)$]



(i) Complete seven additional rows of the table. [7]

Base 1	Base 2	Altitude	Area
8	$8 + 16 \sin 10°$	$8 \sin 10°$	22.1
8	$8 + 16 \sin 20°$	$8 \sin 20°$	42.5

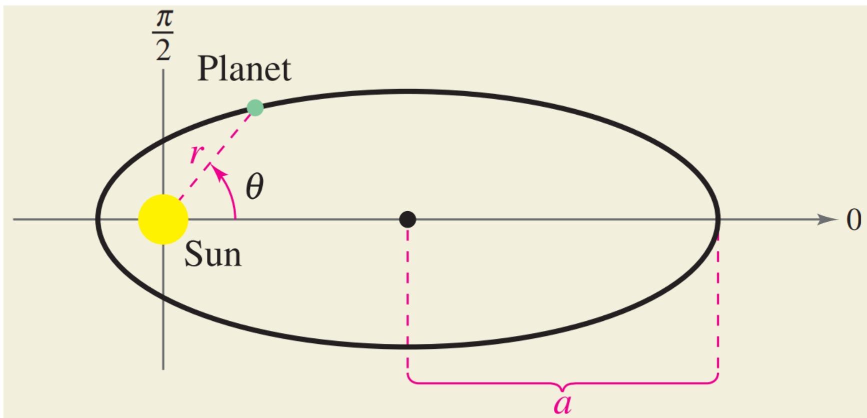
(ii) Use the table to estimate the maximum cross-sectional area. [2]

(iii) Write the area as a function of θ. [6]

B6. (a) The planets travel in elliptical orbits with the sun at one focus. The polar equation of the orbit of a planet with one focus at the pole and major axis of length is

$$r - \frac{(1-e^2)a}{1-e\cos\theta}$$

where e is the eccentricity. The minimum distance (perihelion) from the sun to a planet is $r = a(1-e)$ and the maximum distance (aphelion) is $r = a(1+e)$. For the planet Neptune, $a = 4.495 \times 10^9$ kilometers and $e = 0.0086$. For the dwarf planet Pluto, $a = 5.906 \times 10^9$ kilometers and $e = 0.2488$.



(i) Find the polar equation of the orbit of each planet. [6]

(ii) Find the perihelion and aphelion distances for each planet. [4]

(iii) Is Pluto ever closer to the sun than Neptune?
Until recently, Pluto was considered the ninth planet.
Why was Pluto called the ninth planet and Neptune the eighth planet? [2]

(iv) Do the orbits of Neptune and Pluto intersect?
Will Neptune and Pluto ever collide? Why or why not? [2]

(b) Convert the rectangular equation to polar form. $x^2 + y^2 - 48 = 0$ [6]

END OF QUESTION PAPER

"The English word **science** comes from the Latin word *scio* to know "