GWANDA STATE UNIVERSITY

EMI/EMR: 1101

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF MINING AND METALLURGY

ENGINEERING MATHEMATICS I

EPOCH MINE CAMPUS: FILIBUSI

Mr M NDLOVU

JULY 2019: EXAMINATION
Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

Instruments and Materials

- Calculator.

SECTION A: Answer ALL questions [40].

A1. Determine the derivatives of

(a) $y = 0.34(1-x)e^{0.25x}$ [4]

(b) $y = \dfrac{2}{\theta^2} + 2ln2\theta - 2(\cos 5 + 3\sin 2\theta) - \dfrac{2}{e^{3\theta}}$

and evaluate $\dfrac{dy}{d\theta}$ when $\theta = \dfrac{\pi}{2}$ [6]

A2. Evaluate the following integrals

(a) $\displaystyle\int_2^3 \frac{2x^2+1}{x}dx$ [4]

(b) $\displaystyle\int_0^2 x\sqrt{(2x^2+1)}dx$ [5]

A3. If $\mathbf{A} = 3i - j - 4k$, $\mathbf{B} = -2i + 4j - 3k$, $\mathbf{C} = i + 2j - k$,

(a) What is a unit vector? [2]

(b) Find $\mathbf{B} \cdot \mathbf{C}$ [2]

(c) Find $\mathbf{A} \times \mathbf{B}$ [4]

A4. (a) State the difference between implicit differentiation and partial differentiation. [2]

(b) Solve $\dfrac{1}{x}\dfrac{dy}{dx} + 4y = 2$ given the boundary conditions $x = 0$ when $y = 4$. [6]

(c) Find $\dfrac{\partial^2 z}{\partial y \partial x}$ when, $z = \sqrt{\dfrac{3x}{y}}$ [5]

SECTION B: Answer ANY three questions [60].

B5. (a) Compute the derivative of $y = \dfrac{e^{2x}\cos 3x}{\sqrt{x-4}}$ [6]

(b) A Cadbury Dairy Milk Chocolate bar with a rectangular shape measures 12 centimeters in length, 7 centimeters in width, and 3 centimeters in thickness. Due to escalating costs of cocoa, management decides to reduce the volume of the bar by 15%. To accomplish this reduction, management decides that the new bar should have the same 3 centimeter thickness, but the length and width of each should be reduced an equal number of centimeters.
What should be the dimensions of the new candy bar? [8]

(c) The curve $y = 2x^2 + 3$ is rotated about the $x - axis$ between the limits $x = 0$ and $x = 3$, determine the volume generated. [6]

B6. (a) What are two names given to the method use to find maximum and minimum values of function. [2]

(b) A closed cylindrical container has a surface area of 400 cm^2.
Determine the dimensions for maximum volume. [5]

(c) Show that the differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$
is satisfied when $y = xe^{2x}$ [5]

(d) The velocity constant k of a given chemical reaction is given by:

$$kt = \int \left(\frac{1}{(3-0.4x)(2-0.6x)} \right) dx$$

where $x = 0$ when $t = 0$. Show that: $kt = ln \left\{ \dfrac{2(3-0.4x)}{3(2-0.6x)} \right\}$ [8]

B7. (a) **True** or **False** Integration and Differentiation are ONLY performed when the angle is in radians. [2]

(b) An alternating current, i amperes, is given by $i = 10 \sin 2\pi ft$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20ms$, given that $f = 150$Hz. [5]

(c) The radius of curvature, ρ, of part of a surface when determining the surface tension of a liquid is given by: $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ Find the radius of curvature (correct to 4 significant figures) of the part of the surface having parametric equations
$x = 4\cos^3 t$, $y = 4\sin^3 t$ at $t = \dfrac{\pi}{6}$ rad. [5]

(d) The entropy change $\triangle S$, for an ideal gas is given by:

$$\triangle S = \int_{T_1}^{T_2} C_\nu \frac{dT}{T} - R\int_{V_1}^{V_2} \frac{dV}{V}$$

where T is the thermodynamic temperature, V is the volume and $R = 8.314$. Determine the entropy change when a gas expands from 1 litre to 3 litres for a temperature rise from 100K to 400K given that:

$$C_\nu = 45 + 6\times 10^{-3}T + 8\times 10^{-6}T^2$$

[8]

B8. The profile of a rotor blade is bounded by the lines $x = 0.2$, $y = 2x$, $y = e^{-x}$, $x = 1$ and the $x-axis$. The blade thickness t varies linearly with x and is given by: $t = (1.1-x)$K, where K is a constant.

(a) Sketch the rotor blade, labelling the limits. [4]

(b) Determine, using an iterative method, the value of x, correct to 3 decimal places, where $2x = e^{-x}$ [4]

(c) Calculate the cross-sectional area of the blade, correct to 3 decimal places. [6]

(d) Calculate the volume of the blade in terms of K, correct to 3 decimal places [6]

B9. (a) What is a Limit? [1]

(b) State the fundamental theorem of calculus. [5]

(c) $\int_0^{\pi} 3\sin^3 dx$ use reduction formula. [8]

(d) An equation used in thermodynamics is the Benedict-Webb-Rubine equation of state for the expansion of a gas. The equation is:

$$p = \frac{RT}{V} + \left(B_0RT - A_0 - \frac{C_0}{T^2}\right)\frac{1}{V^2} + (bRT - a)\frac{1}{V^3} + \frac{A\alpha}{V^6} + \frac{C\left(1 + \frac{\gamma}{V^2}\right)}{T^2}\left(\frac{1}{V^3}\right)e^{-\frac{\gamma}{V^2}}$$

Show that $\dfrac{\partial^2 p}{\partial T^2} = \dfrac{6}{V^2T^4}\left[\dfrac{C}{V}\left(1 + \dfrac{\gamma}{V^2}\right)e^{-\frac{\gamma}{V^2}} - C_0\right]$ [6]

END OF QUESTION PAPER

David Hilbert

"Mathematics is a game played according to certain simple rules with meaningless marks on paper."