GWANDA STATE UNIVERSITY

EMI/EMR: 2201

FACULTY OF ENGINEERING AND THE ENVIRONMENT

DEPARTMENT OF MINING AND METALLURGY

APPLIED MATHEMATICS

EPOCH MINE CAMPUS: FILIBUSI

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JUNE 2019: EXAMINATION
Time : 3 hours

Candidates should attempt **ANY FOUR** from this examination paper (25 marks each).

Instruments and Materials

- Scientific Calculator.

- Statistical Tables.

A1. (a) State and explain any three types of error in numerical analysis. [6]

(b) Approximations of $\pi = 3.1159265358979\cdots$ are $\dfrac{22}{7}$ and $\dfrac{355}{113}$.
Determine the corresponding errors and relative errors to 3 significant digits [5]

(c) Determine the limiting relative error of measuring the volume of a cylinder, if $a = 3.14$ is used instead of the number π (take the radius to be 30 cm and the height to be 150 cm) [6]

(d) From time to time, unknown to its employees, the research department at Post Office Bank observes various employees for their work productivity. Recently this department wanted to check whether the four tellers at a branch of this bank serve, on average, the same number of customers per hour. The research manager observed each of the four tellers for a certain number of hours. The following table gives the number of customers served by the four tellers during each of the observed hours The data was analysed and some R package output is shown,

Teller A	Teller B	Teller C	Teller D
19	14	11	24
21	16	14	19
26	14	21	21
24	13	13	26
18	17	16	20
	13	18	

Complete this ANOVA table. [8]

Source	SS	DF	MS	F
Tellers	255.61	...	...	...
Error	...	...	...	
Total	413.82	...		

A2. (a) Determine the truncation error for the formula

$$f' = \frac{1}{3h}[f(x + 5h) - f(x + 2h)]$$

[5]

(b) Use the secant method to find a solution to within 10^{-3} of the positive value x for which

$$f(x) = x^3 - 0.3x^2 - 0.3x - 1.3 = 0$$

[8]

(c) Apply the Runge-Kutta method of the fourth order to find an approximate value of y at $x = 0.2$, in step of 0.1, given the initial value problem :
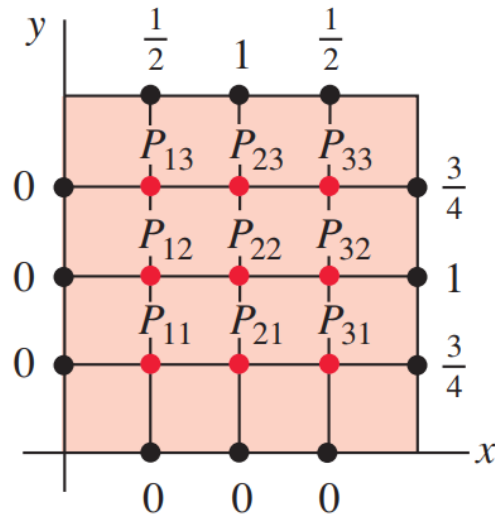
$$y' = x + y^2, y(0) = 1$$

[12]

A3. The boundary-value problem
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \text{ for } 0 < x < 2, \, 0 < y < 2$$

$u(0, y) = 0$, for $u(2, y) = y(2 - y)$, $0 < y < 2$

$u(x, 0) = 0$, for $u(x, 2) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$

yields the following interior point shown in the figure below

(a) What is are the values of n and h? [2]

(b) Using the boundary conditions obtain the nine equations for the interior points. [9]

(c) Write down the banded matrix for these equations. [8]

(d) If that $u_{11} = \frac{7}{64}$, $u_{13} = \frac{145}{448}$, $u_{21} = \frac{51}{224}$, $u_{22} = \frac{13}{32}$, $u_{32} = \frac{135}{224}$, $u_{33} = \frac{39}{64}$
Find u_{12}, u_{23}, u_{31}. [6]

A4. (a) Set up a Newton iteration for computing the square x of a given positive number c and apply it to $c = 2$ [5]

(b) What is interpolation ? when and how is it used in analysis ? [5]

(c) Construct an empirical formula for the function represented by the table below

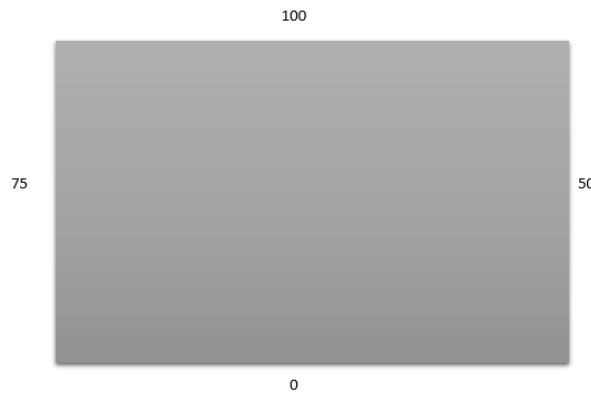| x	0	1	2	3	4	5
y	5.2	8.0	10.4	12.4	14.0	15.2

[8]

(d) Consider the table of values of $y = f(x)$.

x	y
321.0	2.50651
322.8	2.50893
324.2	2.51081
325.0	2.51188

Estimate the value of $f(323.5)$. [7]

A5. (a) It is required to determine the steady state temperature at all points of a heated sheet of metal. The edges of the sheet are kept at a constant temperature: 100, 50, 0, and 75 degrees.

The sheet is divided to 5×5 grids.
Determine the system of equations for this boundary value problem. [10]

(b) Given that $f(x) = xe^x$, use a three-point formula with $h = 0.1$,
to find approximations to $f''(0.5)$.
Compare the calculated value with the true value

$$f''(0.5) = 2.5e^{0.5} \approx 4.121803177$$

[8]

(c) Approximation $\int_0^1 (3x^2 - 4x)dx$ by the trapezoidal formula for $n = 5$.
Evaluate this integral exactly and find the absolute and relative errors
of the result. [7]

A6. (a) State any five merits and five demerits of using MATLAB R2016a. [10]

(b) An article in the *Industrial Hygiene Association Journal* (Vol. 37, 1976, pp. 418-422) describes a field test for detecting the presence of arsenic in the urine samples. The test has been proposed for among the forestry workers because of the increasing use of organic arsenics in the industry. The experiment compared the test as performed by both a trainee and an experienced trainer to an experienced trainer to an analysis at a remote laboratory. Four subjects were selected for testing and are considered as blocks. The response variable is arsenic content (in ppm) in the subject's urine. The data are as follows: Is the a any difference in

	Subjects			
Test	1	2	3	4
Trainee	0.05	0.05	0.04	0.15
Trainer	0.05	0.05	0.04	0.17
Lab	0.04	0.04	0.03	0.10

the arsenic test procedure (compute for both treatments and blocks). [15]

END OF QUESTION PAPER

"Push yourself, because no one else is going to do it for you."